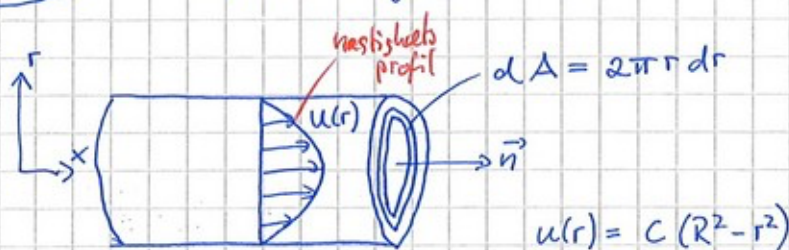


(RÖ3)

(3.01)

Volymflödet genom ett rör!

3.1



Fransida



↖ då $r=R, u=0$
(no-slip)

Ekv. (3.7)

$$Q = \int_A \vec{v} \cdot \vec{n} dA$$

↖ vi behöver ersätta detta

ett area element:

$$dA = 2\pi r dr$$

jag kan säga till er

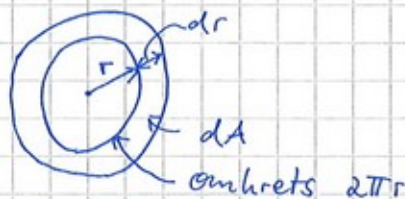
hur kommer man fram till detta?

1) matematiskt: vi letar efter area förändringen med radius

$$A = \pi r^2$$

vi tar
derivatan

$$\frac{dA}{dr} = 2\pi r \Rightarrow dA = 2\pi r dr$$

2) geometriskt:

man kan multiplicera omkretsen med dr :

$$dA = 2\pi r dr$$

(3.7)

$$Q = \int_A \vec{v} \cdot \vec{n} dA = \int_0^R C(R^2 - r^2) \cdot 2\pi r dr$$

hastighet är
samma håll som
 $\vec{n}$, alltså positivt
tecken

$$= \int_0^R 2\pi C (R^2 r - r^3) dr = 2\pi C \left[\frac{R^2 r^2}{2} - \frac{r^4}{4} \right]_0^R$$

$$= 2\pi C \left(\frac{R^4}{2} - \frac{R^4}{4} \right) = 2\pi C \frac{R^4}{4} = \frac{\pi C R^4}{2}$$

3.03Kontinuitets ekv.

3.2

$$\frac{d}{dt} \int_{c.v.} \rho dV + \int_{c.s.} \rho \vec{V} \cdot \hat{n} dA = 0$$

$$\frac{d}{dt} \int_{c.v.} \rho dV = \frac{d}{dt} \left(\rho \cdot \pi \frac{d^2}{4} \cdot h(t) \right) = \frac{\rho \pi d^2}{4} \frac{dh}{dt}$$

Inkompr.

$$\frac{\pi d^2}{4} \frac{dh}{dt} + \underbrace{\int_{c.s.} \vec{V} \cdot \hat{n} dA}_{= Q_1 + Q_2 + Q_3} = 0$$

$$\frac{dh}{dt} = - \frac{4(Q_1 + Q_2 + Q_3)}{\pi d^2}$$

Given: $Q_3 = -0.01 \text{ m}^3/\text{s}$, $V_1 = 3 \text{ m/s}$, $\frac{dh}{dt} = 0$

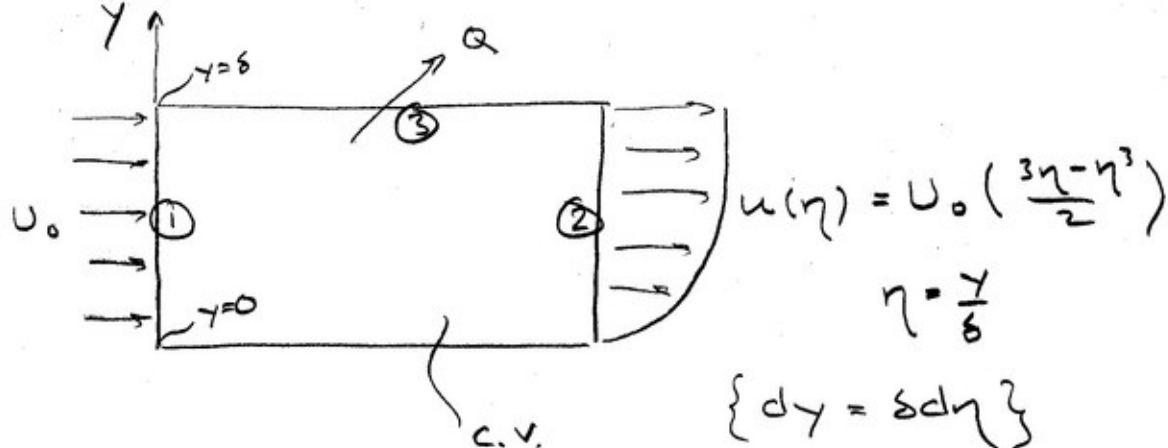
$$\int_{c.s.} \vec{V} \cdot \hat{n} dA = \int_{\textcircled{1}} -V_1 dA + \int_{\textcircled{2}} V_2 dA + Q_3 = 0$$

$$-V_1 A_1 + V_2 A_2 + Q_3 = 0$$

$$V_2 = \frac{V_1 A_1 - Q_3}{A_2} = \frac{3 \cdot \frac{\pi \cdot 0.05^2}{4} - (-0.01)}{\frac{\pi \cdot 0.07^2}{4}} \approx \underline{4.13 \text{ m/s}}$$

3.04

3.3

Kontinuitets ekv.

$$\frac{d}{dt} \int_{c.v.} \rho dV + \int_{c.s.} \rho \vec{V} \cdot \hat{n} dA = 0$$

Fix kontrollvolym, inkompr. fluid

$$\left. \begin{aligned} \frac{d}{dt} \int_{c.v.} \rho dV &= \int_{c.v.} \frac{\partial \rho}{\partial t} dV = 0 \\ \int_{c.s.} \rho \vec{V} \cdot \hat{n} dA &= \rho \int_{c.s.} \vec{V} \cdot \hat{n} dA \end{aligned} \right\} \Rightarrow$$

$$\Rightarrow \int_{c.s.} \vec{V} \cdot \hat{n} dA = 0$$

$$\int_{c.s.} \vec{V} \cdot \hat{n} dA = - \int_{\text{①}} U_0 dA + \int_{\text{②}} u(\eta) dA + Q = 0$$

$$= - \int_0^b U_0 \cdot b \cdot dy + \int_0^1 U_0 \frac{3\eta - \eta^3}{2} b \cdot \delta d\eta + Q = 0$$

$$= - [U_0 b y]_0^b + \left[\frac{U_0}{2} \left(\frac{3}{2} \eta^2 - \frac{\eta^4}{4} \right) b \delta \right]_0^1 + Q = 0$$

$$= -U_0 b \delta + \frac{5 U_0 b \delta}{8} + Q = 0$$

$$Q = \frac{3 U_0 b \delta}{8}$$

3.05

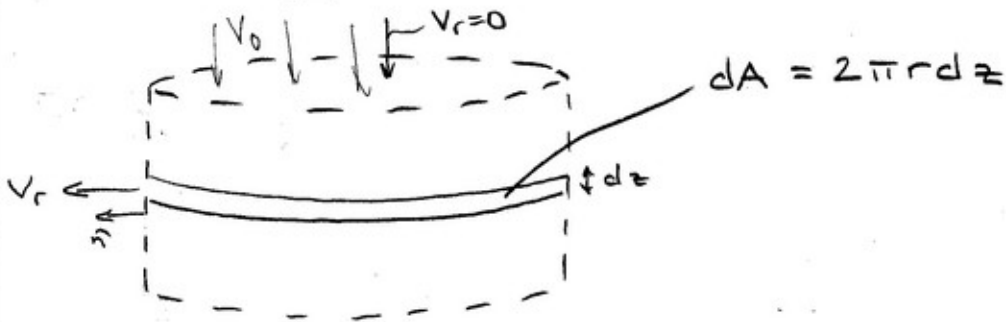
Kont. eqv.Deformerbezog. C.V.

3.4

$$\frac{d}{dt} \int_{c.v.} \rho dV + \int_{c.s.} \rho \vec{V}_r \cdot \hat{n} dA = 0$$

$$\frac{d}{dt} \int_{c.v.} \rho dV = \frac{d}{dt} (\rho \pi r^2 h(t)) = \rho \pi r^2 \frac{dh}{dt} \quad (\text{Inkompr.})$$

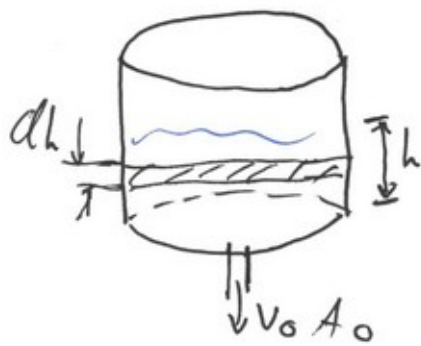
$$\pi r^2 \frac{dh}{dt} + \int_{c.s.} \vec{V}_r \cdot \hat{n} dA = 0, \quad \underline{\frac{dh}{dt} = -V_0}$$



$$\pi r^2 (-V_0) + \int_0^h V_r(r) \cdot 2\pi r dz = \pi r^2 V_0 + V(r) \cdot 2\pi r \cdot h = 0$$

$$V(r) = \frac{V_0 \pi r^2}{2\pi r h} = \underline{\underline{\frac{V_0 r}{2h}}}$$

3.5 / 3.5



known: v_0, h_0

Find: $\frac{dh}{dt}$

Assume: 1D, incomp

Solution Use RTT for conservation of mass (3.20)

$$0 = \frac{d}{dt} \left(\int_{CV} \rho \, dv \right) + \int_{CS} \rho (\vec{v} \cdot \vec{n}) \, dA = 0$$

incomp incomp

$$= \frac{d}{dt} (V) + \underbrace{v_0 \cdot A_0}_{\text{outflow}} = 0 \Rightarrow \frac{d}{dt} (V) = -v_0 A_0 \quad [t]$$

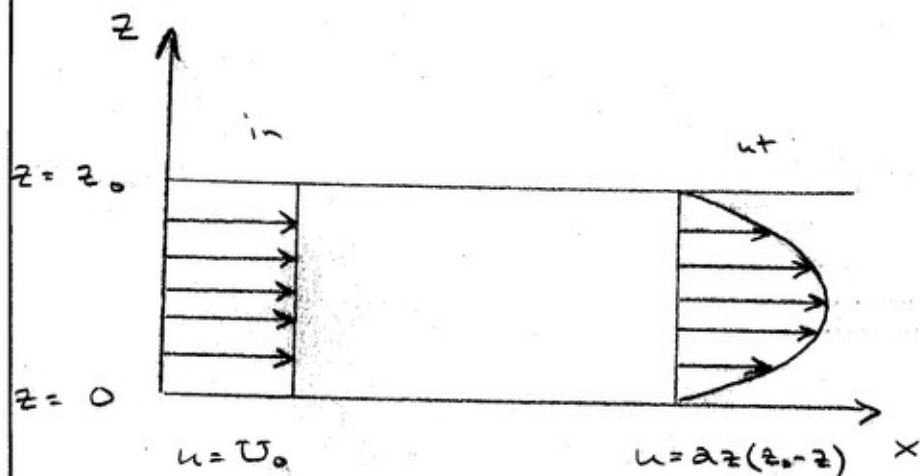
$$V = \frac{\pi d^2}{4} \cdot h$$

$$\Rightarrow \frac{d}{dt} \left(\frac{\pi d^2}{4} \cdot h \right) = -v_0 A_0 \Rightarrow$$

$$\frac{dh}{dt} = - \frac{4 v_0 A_0}{\pi d^2}$$

3.06

3.6



Givet:
 $U_0 = 8 \text{ cm/s}$
 $z_0 = 4 \text{ cm}$

Kontinuitet vid inkompressibel strömning

$$Q_{in} = Q_{ut}$$

Antag djupet i y-led är b.

$$Q_{in} = \int_0^b \int_0^{z_0} U_0 dz dy = z_0 U_0 b$$

$$Q_{ut} = \int_0^b \int_0^{z_0} az(z_0 - z) dz dy = ab \int_0^{z_0} z_0 z - z^2 dz$$

$$= ab \left[\frac{z_0}{2} z^2 - \frac{z^3}{3} \right]_0^{z_0} = \frac{az_0^3}{6} b$$

$$Q_{in} = Q_{ut} \Rightarrow z_0 U_0 b = \frac{1}{6} a z_0^3 b$$

$$a = \frac{6U_0}{z_0^2}$$

Vid utlopp: $u(z) = \frac{6U_0}{z_0^2} z(z_0 - z)$

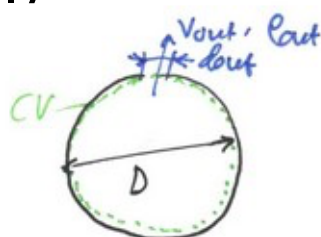
Sök u_{max} :

Hitta z där u är störst: $\frac{du}{dz} = \frac{6U_0}{z_0^2} (z_0 - 2z)$

$$\frac{du}{dz} = 0 \Rightarrow z = \frac{1}{2} z_0 \quad (\text{dvs mitt i kanalen})$$

$$u_{max} = u\left(\frac{1}{2} z_0\right) = \frac{6U_0}{z_0^2} \frac{1}{2} z_0 \left(z_0 - \frac{1}{2} z_0\right) = \frac{3U_0}{2} = \underline{\underline{12 \text{ cm/s}}}$$

P3.7 | 3.7



known:
 $V_{out} = 360 \text{ m/s}$
 $\rho_{out} = 2.5 \text{ kg/m}^3$
 $d_{out} = 0.005 \text{ m}$
 $D = 0.35 \text{ m}$

Task: Derive an expression for the rate of change of average density $d\rho/dt$ in the tank & calculate the value of $d\rho/dt$ for the given data.

Assume: 1D in/out, constant outlet properties, stationary CV $\rightarrow \tilde{V}_r \equiv \tilde{V}$, fix CV

Solution: Use RTT for conservation of mass to derive $\frac{d\rho}{dt}$.

$$(3.20) \quad \frac{d}{dt} (\int_{CV} \rho dV) + \int_{CS} \rho (\tilde{V}_r \cdot \tilde{n}) dA = 0 \quad , \quad \tilde{V}_r = \tilde{V} \text{ (stationary CV)} =$$

$$= \frac{d}{dt} (\underbrace{\rho V_{CV}}_{\text{fix CV}}) + \underbrace{\rho_{out} V_{out} \pi \frac{d_{out}^2}{4}}_{\text{1D in/out}} = 0$$

Volume of the CV given by the sphere volume: $V_{CV} = \frac{1}{6} \pi D^3$ } $\Rightarrow$

$$\frac{d}{dt} \left(\rho \frac{1}{6} \pi D^3 \right) = - \rho_{out} V_{out} \pi \frac{d_{out}^2}{4} \Rightarrow$$

$$\frac{d\rho}{dt} = \frac{\rho_{out} V_{out} \pi \frac{d_{out}^2}{4}}{\frac{1}{6} \pi D^3} = - \frac{3}{2} \frac{\rho_{out} V_{out} d_{out}^2}{D^3}$$

b) Calculate the value of $\frac{d\rho}{dt} = \left\{ \begin{matrix} V_{out} = 360 \text{ m/s} \\ \rho_{out} = 2.5 \text{ kg/m}^3 \\ d_{out} = 0.005 \text{ m} \end{matrix} \middle| D = 0.35 \text{ m} \right\} = -0.79 \frac{\text{kg}}{\text{m}^3 \cdot \text{s}}$

known:

$$Q_1 = 0,5 \text{ m}^3/\text{s}$$

$$V_2 = 12 \text{ m/s}$$

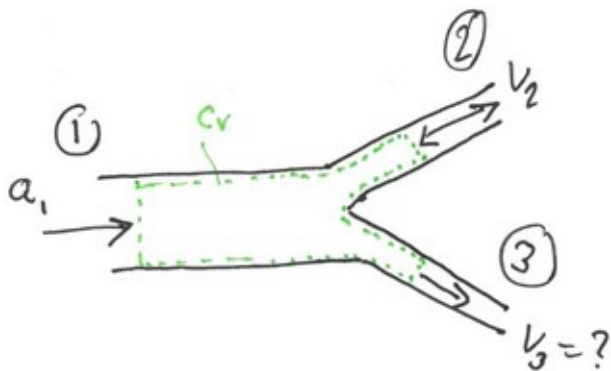
$$d_2 = 0,18 \text{ m}$$

$$d_3 = 0,13 \text{ m}$$

$$\rho = 680 \text{ kg/m}^3 \quad (\text{Table A3})$$

Task: a) Calculate the velocity at (3)

b) is the flow going in or out?



Assume: Steady-state, stationary CV, 1D in/out, incompressible
 $\left(\frac{d}{dt} = 0\right) \quad (\tilde{V}_r \equiv \tilde{V}) \quad (\rho = \text{constant})$

Solution: Use RTT for conservation of mass and calculate V_2

$$(3.20) \quad \underbrace{\frac{d}{dt} \left(\int_{CV} \rho dV \right)}_{\frac{d}{dt} = 0, \text{ steady state}} + \underbrace{\int_{CS} \rho (\tilde{V}_r \cdot \tilde{n}) dA}_{\tilde{V}_r = \tilde{V}, \text{ stationary}} = 0 \Rightarrow \int_{CS} \rho (\tilde{V} \cdot \tilde{n}) dA = 0$$

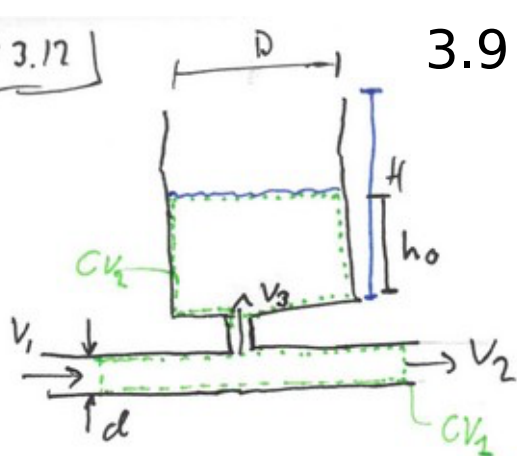
$$1\text{D in/out gives: } \underbrace{\sum (\rho_i A_i V_i)_{\text{out}}}_{\text{incomp.}} - \underbrace{\sum (\rho_i A_i V_i)_{\text{in}}}_{\text{incomp.}} = 0 \quad \Rightarrow$$

Volume flow defined as: $Q = V \cdot A$

$$-Q_1 + \underbrace{Q_2}_{\text{assume outflow}} + Q_3 = 0 \Rightarrow Q_3 = Q_1 - V_2 A_2 = Q_1 - V_2 \pi \frac{d_2^2}{4} = +0,19 \dots \frac{\text{m}^3}{\text{s}}$$

$\therefore Q_2 = 0,19 \dots \frac{\text{m}^3}{\text{s}}$, the positive sign show that Q_2 is flowing out of CV. (b)

$$V_3 = \frac{Q_3}{A_3} = \frac{4Q_3}{\pi d_3^2} = 14,7 \text{ m/s} \quad (2)$$



known:

$$\begin{aligned} V_1 &= 2.5 \text{ m/s} & d &= 0.12 \text{ m} & H &= 1 \text{ m} \\ V_2 &= 1.9 \text{ m/s} & D &= 0.75 \text{ m} & h_0 &= 0.3 \text{ m (at } t=0) \end{aligned}$$

Task: Calculate the time to fill the tank

Assume: CV1: steady-state, 1D in/out, incomp. stationary
CV2: unsteady, ———— 11 ————

Solution: Use RTT for conservation of mass for CV1 to calculate the flow into CV2 (the tank). → Use RTT for mass conservation of CV2 to find the time.

$$\text{RTT CV}_1 \text{ (3.20): } \frac{d}{dt} \left(\int_{CV1} \rho dV \right) + \int_{CS1} \rho (\tilde{\mathbf{V}} \cdot \tilde{\mathbf{n}}) dA = 0 \Rightarrow [1D \text{ in/out}] \Rightarrow$$

incomp
 $\rho = \rho_0$
 $\tilde{\mathbf{V}}_r = \tilde{\mathbf{V}}$

$$-Q_1 + Q_2 + Q_3 = 0 \Rightarrow Q_3 = Q_1 - Q_2 = [Q = V \cdot A = V \frac{\pi d^2}{4}] \Rightarrow$$

$$Q_3 = \frac{\pi d^2}{4} (V_1 - V_2) = 6.785 \cdot 10^{-3} \frac{\text{m}^3}{\text{s}}, \text{ Flow in into the tank}$$

$$\text{RTT CV}_2 \text{ (3.20): } \frac{d}{dt} \left(\int_{CV2} \rho dV \right) + \int_{CS2} \rho (\tilde{\mathbf{V}} \cdot \tilde{\mathbf{n}}) dA = 0 =$$

$$= \frac{d}{dt} (V) - Q_3 = 0 \Rightarrow \frac{d}{dt} (V) = Q_3 \quad [A]$$

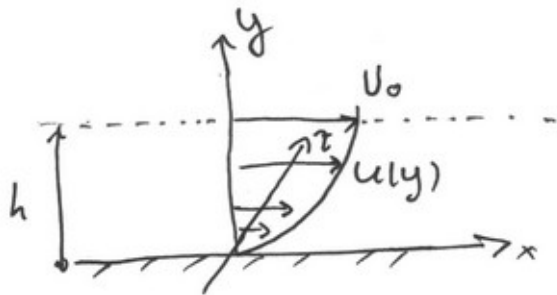
incomp
 $\rho = \rho_0$
 $\tilde{\mathbf{V}}_r = \tilde{\mathbf{V}}$
1D in/out, negative since Q3 is in

The volume V is: $V = \frac{\pi D^2}{4} \cdot h$ [where $h = h_0$ at $t = 0$ (B.C)] in [A]

$$\frac{d}{dt} \left(\frac{\pi D^2}{4} h \right) = Q_3 \Rightarrow \frac{dh}{dt} = \frac{4Q_3}{\pi D^2} \rightarrow \text{Integrate this to find } t$$

$$\int_{h_0}^H dh = \int_0^t \frac{4Q_3}{\pi D^2} dt \Rightarrow H - h_0 = \frac{4Q_3}{\pi D^2} [t - 0] \Rightarrow t = \frac{(H - h_0) \pi D^2}{4Q_3} = 45.6 \text{ s}$$

p3.26 3.10



known

$$u(y) = U_0 \left(\frac{2y}{h} - \frac{y^2}{h^2} \right)$$

$$h = 13 \text{ mm} = 0.013 \text{ m}$$

$$Q/b = 5 \text{ L/min/m (per unit width)} = \frac{5 \cdot 10^{-3} \text{ m}^3}{60 \text{ s}} \frac{\text{m}}{\text{m}}$$

b = width of channel

Task: Find U_0 in mm/s

Assume: Steady-state, incompressible

Solution: Use the definition of volumetric flow rate (3.7)

$$(3.7): Q = \int (\vec{v} \cdot \vec{n}) dA = \int_0^b \int_0^h u(y) dy dz = b \int_0^h u(y) dy = Q \Rightarrow$$

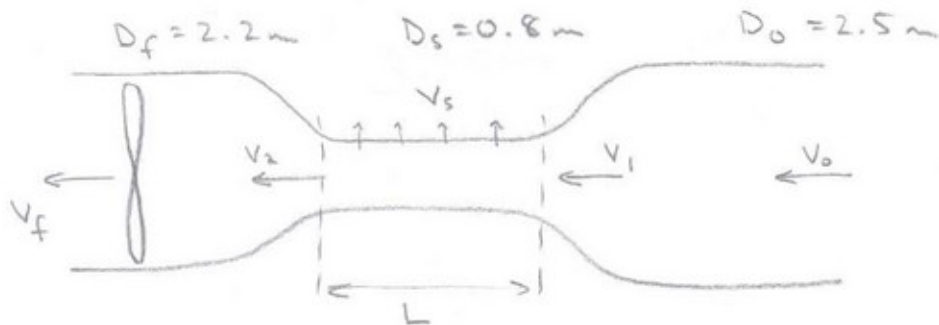
$$\frac{Q}{b} = \int_0^h u(y) dy = \int_0^h U_0 \left(\frac{2y}{h} - \frac{y^2}{h^2} \right) dy = \frac{U_0}{h} \left[y^2 - \frac{y^3}{3h} \right]_0^h$$

$$\frac{Q}{b} = \frac{2}{3} U_0 h \Rightarrow \text{Solve for } U_0$$

$$U_0 = \frac{3}{2h} \cdot \left(\frac{Q}{b} \right) = \frac{3}{2 \cdot 0.013} \cdot \left(\frac{5 \cdot 10^{-3}}{60} \right) = 9.6 \text{ mm/s}$$

3.11

3.11



Antal h_{z1}: 1200/m²

D_{h_{z1}} = 5 mm

V_{h_{z1}} = 8 m/s

Antag inkompressibelt

a) sök V_o

Kontinuitet ger: Q_o = Q₁

$$A_o V_o = A_1 V_1$$

$$\frac{\pi D_o^2}{4} V_o = \frac{\pi D_s^2}{4} V_1$$

$$V_o = \frac{D_s^2}{D_o^2} V_1 \approx \underline{\underline{3.58 \text{ m/s}}}$$

b) sök V₂

Kontinuitet ger: Q₁ = Q₂ + Q_{h_{z1}}

$$A_1 V_1 = A_2 V_2 + A_{h_{z1}} V_{h_{z1}}$$

$$\text{antal } h_{z1} = \pi D_s L \cdot 1200 \approx 12064 \text{ st}$$

$$\Rightarrow \frac{\pi D_s^2}{4} V_1 = \frac{\pi D_s^2}{4} V_2 + \frac{\pi D_{h_{z1}}^2}{4} \cdot 12064 \cdot V_{h_{z1}}$$

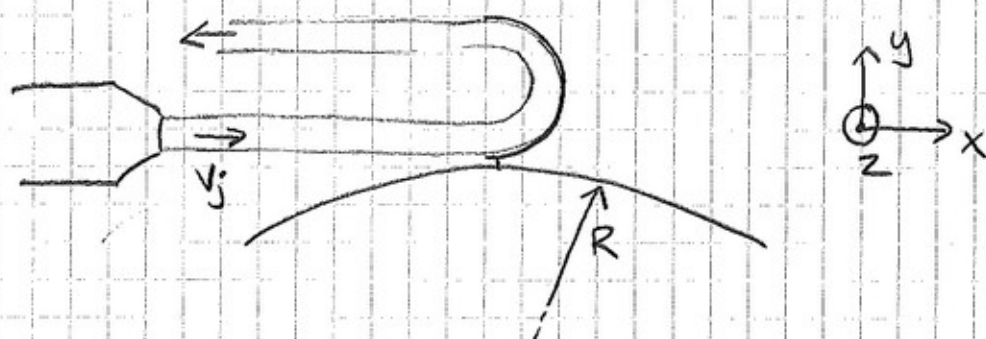
$$V_2 = \frac{D_s^2 V_1 - 12064 D_{h_{z1}}^2 V_{h_{z1}}}{D_s^2} \approx \underline{\underline{31.2 \text{ m/s}}}$$

c) sök V_f, kontinuitet ger: V_f = $\frac{D_s^2}{D_f^2} V_2$

$$V_f \approx 4.13 \text{ m/s}$$

3.13 3.12 (1)

Strålens area = A_j , densitet = ρ



Deluppgift 1:

> Härled ett uttryck för effekten P som strålen överför till hjulet som funktion av indaten ovan.

Bestäm kontrollvolym!



- Kontrollvolymen är rörlig och rör sig med turbinhjulet. $V_{cv} = \Omega R$
- Vi antar alltså att den translaterar linjärt (=ingen acceleration)
- In- och utlopp kan antas 1-dimensionella

Notera; Summan av krafterna på kontrollvolymen (här $\sum F_x$) måste utöva en motsatt kraft på omgivningen (här F_s),

dvs: $F_s = -\sum F_x$

Effekten kan bestämmas genom:

$$P = F_s \cdot \Omega R \quad [A]$$

Utgå från impulssatsen:

$$\frac{d}{dt}(mV)_{\text{syst}} = \sum F = \frac{d}{dt} \left(\int_{cv} V \rho dV \right) + \int_{cs} V \rho (V_r \cdot n) dA \quad (3.35)$$

Hur kan vi förenkla?

Vätskestråle $\rightarrow$ antag inkompressibelt $\rightarrow \rho$ konstant

Dessutom kan vi anta att flödet är stationärt, inte heller

3.12 (2)

ändrar kontrollvolymen sig i storlek

$$\Rightarrow \frac{d}{dt} \left(\int_{\omega} \rho dV \right) = 0 \quad \text{dvs;}$$

$$\Sigma F = \int_{CS} \rho (\mathbf{V}_r \cdot \mathbf{n}) dA$$

Vi har bara flöde över två ytor ("in" och "ut") :

$$\Sigma F = \int_{in} \rho (\mathbf{V}_{r,in} \cdot \mathbf{n}_{in}) dA + \int_{ut} \rho (\mathbf{V}_{r,ut} \cdot \mathbf{n}_{ut}) dA$$

Notera att detta är en vektor-relation som med vektorerna utskrivna ser ut som;

$$\begin{bmatrix} \Sigma F_x \\ \Sigma F_y \\ \Sigma F_z \end{bmatrix} = \int_{in} \begin{bmatrix} V_{in,x} \\ 0 \\ 0 \end{bmatrix} \rho \begin{bmatrix} V_{r,in,x} \\ 0 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} dA + \int_{ut} \begin{bmatrix} V_{ut,x} \\ 0 \\ 0 \end{bmatrix} \rho \begin{bmatrix} V_{r,ut,x} \\ 0 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} dA [B]$$

Ovan har jag använt mig av det faktum att vi inte har några hastigheter, vare sig av strölen eller kontrollvolymen, i y- eller z-riktningen.

Normalvektorn till kontrollvolymens yta är definierad som positiv ut från kontrollvolymen vilket här innebär att

$$\mathbf{n}_{in} = \mathbf{n}_{ut} = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}$$

För att räkna ut ΣF_x behöver vi uttrycka $V_{in,x}$, $V_{r,in,x}$, $V_{ut,x}$ och $V_{r,ut,x}$ i kända variabler. Vi vet att

$$V_{in,x} = V_j$$

Relativhastigheter ges av :

$$\mathbf{V}_r = \mathbf{V} - \mathbf{V}_{cv} \quad (3.14)$$

Här är $V_{cv,x}$ (kontrollvolymens hastighet i x-riktning) = ΩR

→

$$V_{r,in,x} = V_{in,x} - V_{cv,x} = V_j - \Omega R$$

3.12 (3) För att uttrycka ut-hastigheterna använder vi oss av kontinuitetslikvationen:

$$0 = \frac{d}{dt} \left(\int_V \rho dV \right) + \int_{cs} \rho (\mathbf{V}_r \cdot \mathbf{n}) dA \quad (3.20)$$

Inkompressibelt + att kontrollvolymen inte ändrar sig i storlek:

$$0 = \int_{cs} \rho (\mathbf{V}_r \cdot \mathbf{n}) dA$$

Utskrivet för våra två ytor:

$$0 = \int_{in} \rho \begin{bmatrix} V_{r,in,x} \\ 0 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} dA + \int_{ut} \rho \begin{bmatrix} V_{r,ut,x} \\ 0 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} dA$$

i x-led med $V_{r,in,x} = V_j - \Omega R$ och $A_{in} = A_{ut} = A_j$:

$$- \rho (V_j - \Omega R) A_j - \rho V_{r,ut,x} A_j = 0 \Rightarrow$$

$$V_{r,ut,x} = -(V_j - \Omega R) = (\Omega R - V_j)$$

mha ekv 3.14 får vi:

$$V_{r,ut,x} = V_{ut,x} - V_{\omega}$$

$$V_{ut,x} = V_{r,ut,x} + V_{\omega} = - (V_j - \Omega R) + \Omega R = -V_j + 2\Omega R$$

Insättning i [B] ger (endast x-led utskrivet):

$$\begin{aligned} \Sigma F_x &= V_j \rho (V_j - \Omega R) \cdot (-1) A_j + (-V_j + 2\Omega R) \rho (\Omega R - V_j) \cdot (-1) A_j \\ &= -V_j \rho A_j (V_j - \Omega R) + (-V_j + 2\Omega R) \rho A_j (V_j - \Omega R) \\ &= \rho A_j (V_j - \Omega R) (-V_j - V_j + 2\Omega R) \\ &= -2 \rho A_j (V_j - \Omega R)^2 \end{aligned}$$

Som förklarat ovan; $F_s = -\Sigma F_x = 2 \rho A_j (V_j - \Omega R)^2$

Insättning i [A] ger:

$$P = F_s \cdot \Omega R = \underline{\underline{2 \rho A_j \Omega R (V_j - \Omega R)^2}}$$

3.12 (4) Deluppgift 2:

Vid vilken vinkelhastighet överförs maximal effekt?

$P(\Omega)$ och då $\frac{dP}{d\Omega} = 0$ och $\frac{d^2P}{d\Omega^2} < 0$ för $P_{\max}$

Förenkla P för derivering:

$$P = 2\rho A_j \Omega R (V_j - \Omega R)^2 = 2\rho A_j R (V_j^2 \Omega - 2V_j R \Omega^2 + R^2 \Omega^3)$$

$$\frac{dP}{d\Omega} = 2\rho A_j R (V_j^2 - 4V_j R \Omega + 3R^2 \Omega^2) = 0 \Rightarrow$$

$$V_j^2 - 4V_j R \Omega + 3R^2 \Omega^2 = 0$$

$$\Omega^2 - \frac{4V_j}{3R} \Omega + \frac{V_j^2}{3R^2} = 0$$

$$\Omega = \frac{2V_j}{3R} \pm \sqrt{\frac{4}{9}\left(\frac{V_j}{R}\right)^2 - \frac{1}{3}\left(\frac{V_j}{R}\right)^2} = \frac{2V_j}{3R} \pm \frac{1V_j}{3R}$$

$\Omega_1 = \frac{V_j}{R}$ (minimal effekt, stäla och turbinhjul rör sig lika snabbt)

$$\Omega_2 = \frac{1}{3} \frac{V_j}{R}$$

Dubbelkolla att Ω_2 är ett maximum

$$\frac{d^2P}{d\Omega^2} = 2\rho A_j R (-4V_j R + 6R^2 \Omega)$$

$$\Omega_2 \text{ ger } \frac{d^2P}{d\Omega^2} < 0$$

$$P_{\max} = 2\rho A_j \cdot \frac{1}{3} \frac{V_j}{R} R \left(V_j - \frac{1}{3} \frac{V_j}{R} R \right)^2 = \dots = \underline{\underline{\frac{8}{27} \rho A_j V_j^3}}$$

3.13 (1) RTT för $m\bar{V}$

$$\frac{d}{dt}(m\bar{V})_{\text{sys}} = \sum \bar{F} = \underbrace{\frac{d}{dt} \int_{\text{c.v.}} \bar{V} \rho dV}_{\text{c.v.}} + \underbrace{\int_{\text{c.s.}} \bar{V} \rho (\bar{V}_r \cdot \hat{n}) dA}_{\text{c.s.}}$$

$$\begin{cases} \sum F_x = -D \cdot b \\ \sum F_y = 0 \\ \sum F_z = 0 \end{cases}$$

Fix C.V. : $\bar{V}_r = \bar{V}$

Inkompr. : $\frac{\partial \rho}{\partial t} = 0$

Stationärt : $\frac{\partial u}{\partial t} = \frac{\partial v}{\partial t} = \frac{\partial w}{\partial t} = 0$

$$-Db = \int_{\text{c.s.}} u \rho (\bar{V} \cdot \hat{n}) dA =$$

$$= \underbrace{\int_0^h U_0 \rho (-U_0) dA}_{\text{①}} + \underbrace{\int_0^b u \rho u dA}_{\text{②}} = \left\{ dA = b dy \right\} =$$

$$= \int_0^h (-\rho U_0^2) b dy + \int_0^b \rho (u(y))^2 b dy \quad (*)$$

För att finna h , RTT för m

$$\left(\frac{dm}{dt} \right)_{\text{sys}} = 0 = \underbrace{\frac{d}{dt} \int_{\text{c.v.}} \rho dV}_{\text{c.v.}} + \underbrace{\int_{\text{c.s.}} \rho (\bar{V}_r \cdot \hat{n}) dA}_{\text{c.s.}}$$

Inkompr., fix C.V., stationärt :

$$0 = \int_{\text{c.s.}} \rho \bar{V} \cdot \hat{n} dA = \underbrace{\int_0^h \rho (-U_0) dA}_{\text{①}} + \underbrace{\int_0^b \rho u(y) dA}_{\text{②}} \Rightarrow$$

$$\Rightarrow \int_0^h \rho U_0 b dy = \int_0^b \rho u(y) b dy \quad (**)$$

3.15

forte.

(**) in (*):

3.13 (2)

$$-U_0 \int_0^h \rho U_0 b dy + \int_0^{\delta} \rho (u(y))^2 b dy = -D b$$

$$= \int_0^{\delta} \rho u(y) b dy$$

$$\Rightarrow \int_0^{\delta} (-\rho U_0 u(y)) dy + \int_0^{\delta} \rho u(y) u(y) dy = -D$$

$$\rho \int_0^{\delta} u(y) [u(y) - U_0] dy = -D$$

$$D = \rho \int_0^{\delta} u(y) [U_0 - u(y)] dy$$

Momentum
integral
theory (4.7)

$$D = \rho \int_0^{\delta} U_0^2 \sin^2\left(\frac{\pi y}{2\delta}\right) - U_0^2 \sin^2\left(\frac{\pi y}{2\delta}\right) dy =$$

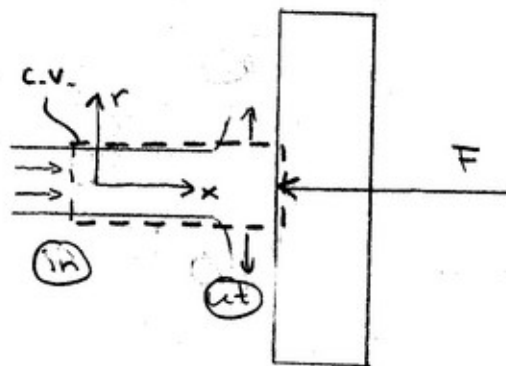
$$= \rho U_0^2 \left[-\frac{2\delta}{\pi} \cos\left(\frac{\pi y}{2\delta}\right) - \left(\frac{y}{2} - \frac{\sin\left(\frac{\pi y}{\delta}\right)}{\left(\frac{2\pi y}{\delta}\right)}\right) \right]_0^{\delta} =$$

$$= \rho U_0^2 \left[-\frac{\delta}{2} + \frac{2\delta}{\pi} \right] = \rho U_0^2 \delta \left(\frac{2}{\pi} - \frac{1}{2} \right) =$$

$$= 998 \cdot 3^2 \cdot 2 \cdot 10^{-3} \left(\frac{2}{\pi} - \frac{1}{2} \right) \approx 2.45 \text{ N/m}$$

3.17

3.14



Søk F genom kraftbalans:

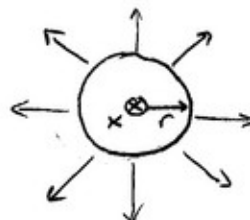
Impulssatsen: Reynolds transport theorem for $\begin{cases} B = m\bar{V} \\ \rho = \bar{\rho} \end{cases}$

$$\sum \bar{F} = \frac{d}{dt} (m\bar{V})_{sys} = \underbrace{\frac{d}{dt} \int_{cv} \bar{V} \rho dV}_{=0} + \int_{cs} \bar{V} \rho (\bar{V}_r \cdot \hat{n}) dA$$

enter stationært
 fix c.v.
 inkompressibelt

$$\sum \bar{F} = \int_{in} V_j \hat{x} \rho (-V_j) dA + \int_{ut} V_{ut} \hat{r} V_{ut} dA$$

utloppet $= 0$ ger
 krefter som tær
 ut vandrere

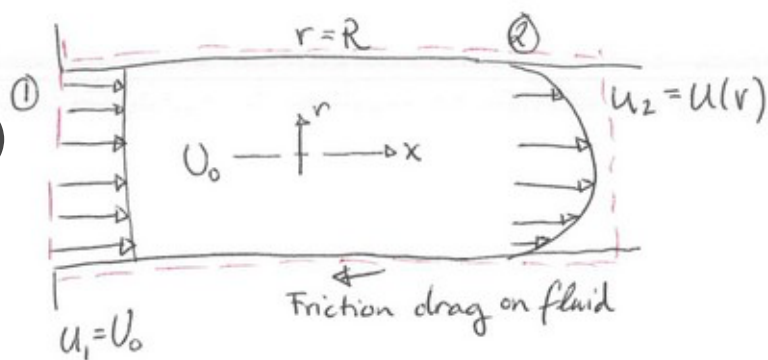


$$\sum \bar{F} = -F \hat{x} = \int_{in} -\rho V_j^2 \hat{x} dA$$

$$F = \frac{\pi D_j^2}{4} \rho \int V_j^2 = \frac{\pi \cdot 0.10^2}{4} \cdot 998 \cdot 8^2 \approx \underline{\underline{502N}}$$

3.15

3.15 (1)



Laminar:
 $u_2 = U_{\max} \left(1 - \frac{r^2}{R^2}\right)$

Turbulent:
 $u_2 = U_{\max} \left(1 - \frac{r}{R}\right)^{1/7}$

Find the wall drag force, $F(p_1, p_2, \rho, U_0, R)$ for

a) laminar flow

b) turbulent flow

Reynolds transport theorem for linear momentum:

$$\sum \vec{F} = \underbrace{\frac{d}{dt} \int_{CV} \vec{V} \rho dV}_{=0 \text{ steady state incompressible}} + \int_{CS} \vec{V} \rho (\vec{V}_r \cdot \vec{n}) dA$$

Only forces in x-direction so we will look at x-component only

$$\sum F_x = \underbrace{(P_1 - P_2) \pi R^2}_{\substack{\text{cross-section} \\ \text{area} \\ \text{the pressure diff.} \\ \text{driving the flow}}} - F_{\text{drag}}$$

Two control surfaces: (again only looking at x-component)
 note fix CV, no $\vec{V}_r$

$$\int_{CS} \vec{V}_x \rho (\vec{V}_x \cdot \vec{n}) dA = \int_{(1)} U_0 \rho (-U_0) dA + \int_{(2)} u_2 \rho u_2 dA$$

$$\Rightarrow \sum F_x = - \int_{(1)} U_0^2 \rho dA + \int_{(2)} u_2^2 \rho dA = [\text{1D inlet}] = -\pi R^2 \rho U_0^2 + \rho \int_{(2)} u_2^2 dA$$

incompressible can be moved out of integr

$$\Rightarrow (P_1 - P_2) \pi R^2 - F_{\text{drag}} = -\pi R^2 \rho U_0^2 + \rho \int_{(2)} u_2^2 dA$$

$$\Rightarrow F_{\text{drag}} = \pi R^2 (P_1 - P_2 + \rho U_0^2 - \rho \int_{(2)} u_2^2 dA)$$

3.15 (2)

a) laminar flow:

We will use the momentum flux correction (see lecture 5)
(note that this is a very small part of the course and not even in the FS)

$$\rho \int u_z^2 dA = \beta V_{ave}^2 \rho A = \beta V_{ave} \dot{m}$$

From continuity $Q_1 = Q_2 \Rightarrow V_{ave} = U_{2,ave} = U_0$

$$\Rightarrow \rho \int u_z^2 dA = \beta U_0^2 \rho \pi R^2 = \frac{4 U_0^2 \rho \pi R^2}{3}$$

← OBS! find in book or lecture slides.

where the momentum flux correction $\beta = \frac{4}{3}$ for laminar flow.

$$\Rightarrow F_{drag} = \pi R^2 (P_1 - P_2 - \frac{1}{3} \rho U_0^2)$$

b) turbulent flow

$\beta = 1.02$ for turbulent flow with $1/7$ as exponent of velocity profile

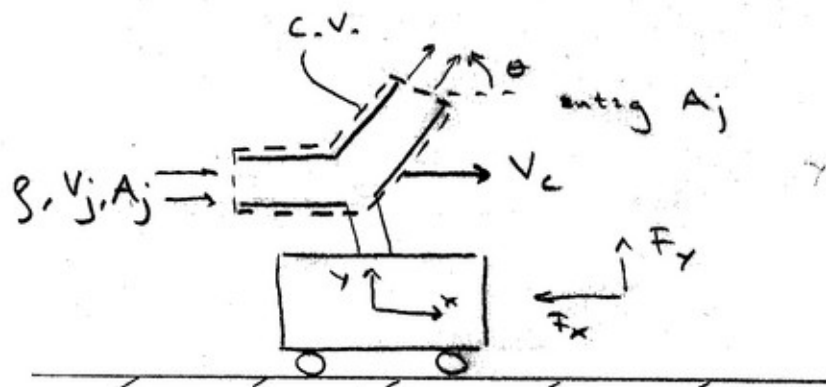
$$\rho \int u_z^2 dA = \beta U_0^2 \rho \pi R^2 = 1.02 U_0^2 \rho \pi R^2$$

$$\Rightarrow F_{drag} = \pi R^2 (P_1 - P_2 - 0.02 \rho U_0^2)$$

We could solve this by integration but we would get complicated integrals and a function dependent on U_{max} which was not asked for. Using the momentum flux correction is very convenient as we only need to know the average velocity and not the maximum.

3.18

3.16 (1)



Låt koord.
systemet
följa vagnen!
 $\Rightarrow \bar{V} = \bar{V}_r$
i impulsatsen!

REYNOLDS TRANSPORT TEOREM $\begin{cases} B = m \bar{V} \\ B = \bar{V} \end{cases}$

$\Rightarrow$ IMPULSSATSEN!

$$\sum \vec{F} = \underbrace{\frac{d}{dt} \int_{C.V.} \rho \bar{V} dV}_{=0 \text{ ty}} + \int_{C.S.} \rho \bar{V} (\bar{V}_r \cdot \hat{n}) dA$$

$\hat{n}$

inkomp.
stationært
färdig men ej
deformerbar C.V.

$$\hat{n}_{in} = -\hat{x}$$

$$\hat{n}_{ut} = \cos \theta \hat{x} + \sin \theta \hat{y}$$

$$|V_{c, in}| = V_j - V_c$$

Kontinuitet ger $V_{ut} = V_{in} = V_j$

$$|V_{r, ut}| = V_j - V_c$$

$$\bar{V}_{r, in} = (V_j - V_c) \hat{x}$$

$$\bar{V}_{r, ut} = (V_j - V_c) \cos \theta \hat{x} + (V_j - V_c) \sin \theta \hat{y}$$

3.18

forts.

3.16 (2)

$$\Sigma \vec{F} = -F_x \hat{x} + F_y \hat{y} = \int \rho \vec{V} (\vec{V}_r \cdot \hat{n}) dA + \int \rho \vec{V} (\vec{V}_r \cdot \hat{n}) dA$$

$\vec{V} = \vec{V}_r$ ty koord. sys. følger vagnen!

$$-F_x \hat{x} + F_y \hat{y} = \int \rho \underbrace{(V_j - V_c) \hat{x}}_{\vec{V}_r} \underbrace{(V_j - V_c) \hat{x}}_{\vec{V}_r} \cdot \underbrace{(-\hat{x})}_{\hat{n}} dA +$$

$$+ \int \rho \underbrace{(V_j - V_c)(\cos \theta \hat{x} + \sin \theta \hat{y})}_{\vec{V}_r} \underbrace{(V_j - V_c)(\cos \theta \hat{x} + \sin \theta \hat{y})}_{\vec{V}_r} \cdot \underbrace{(\cos \theta \hat{x} + \sin \theta \hat{y})}_{\hat{n}} dA$$

$$= A_j \rho (V_j - V_c)(V_c - V_j) \hat{x} + A_j \rho (V_j - V_c)^2 (\cos \theta \hat{x} + \sin \theta \hat{y})$$

$$= \rho A_j (V_j - V_c)^2 [(\cos \theta - 1) \hat{x} + \sin \theta \hat{y}]$$

$$\begin{cases} a) & F_x = \rho A_j (V_j - V_c)^2 (1 - \cos \theta) \\ & F_y = \rho A_j (V_j - V_c)^2 \sin \theta \\ & \vec{F} = -F_x \hat{x} + F_y \hat{y} \end{cases}$$

b) Effekt till vagnen:

$$\dot{P} = \underline{V_c \cdot F_x} = \rho A_j V_c (V_j - V_c)^2 (1 - \cos \theta)$$

endast hastighet
i x-riktning!

$$|\vec{F}| = \rho A_j (V_j - V_c)^2 [(1 - \cos \theta)^2 + \sin^2 \theta]^{1/2}$$

3.18
forts.

c) Sök $|\vec{F}|_{\max}$

$$3.16 (3) \quad |\vec{F}| = g A_j (V_j - V_c)^2 \underbrace{\sqrt{(\cos\theta - 1)^2 + \sin^2\theta}}_{= 2 - 2\cos\theta} = \sqrt{(-F_x)^2 + F_y^2}$$

$$|\vec{F}| = g A_j (V_j - V_c)^2 \sqrt{2 - 2\cos\theta}$$

$|\vec{F}|$ är maximalt då $V_c = 0$

ty det ger max av $(V_j - V_c)^2$

d) Sök $P_{\max}$

$$P = g A_j V_c (V_j - V_c)^2 (1 - \cos\theta)$$

$$\frac{dP}{dV_c} = g A_j (1 - \cos\theta) V_c (V_j^2 - 2V_j V_c + V_c^2)$$

$$= C \cdot (V_c^3 - 2V_j V_c^2 + V_j^2 V_c)$$

$$\frac{dP}{dV_c} = 3C V_c^2 - 4C V_j V_c + C V_j^2$$

$$P_{\max} \quad \text{fås då} \quad \frac{dP}{dV_c} = 0$$

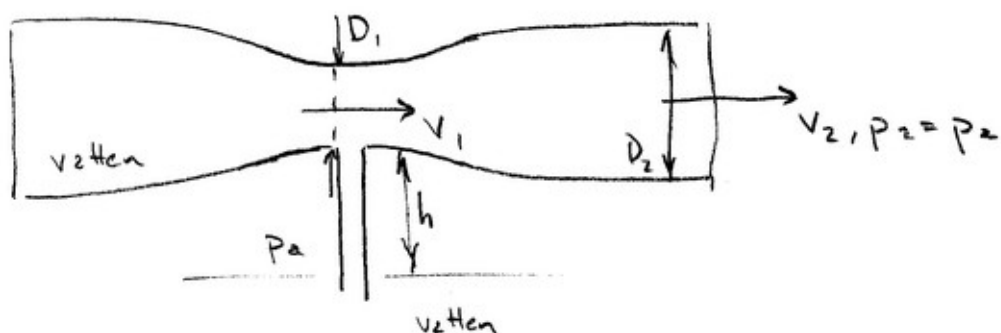
$$\frac{dP}{dV_c} = 0 \Rightarrow V_c' = \frac{2V_j}{3} \pm \sqrt{\frac{4V_j^2}{9} - \frac{1}{3}V_j^2} = V_j \left(\frac{2}{3} \pm \frac{1}{3} \right)$$

$$V_c = V_j \Rightarrow P = 0$$

$$\underline{V_c = \frac{1}{3} V_j \Rightarrow \text{maximal effekt } P_{\max}}$$

3.21

3.17



Hydrostatiska spänningstillståndet:

$$P_1 - P_a = -\rho g h \Rightarrow P_1 = P_a - \rho g h$$

Bernoullis utvidgade ekv:

$$P_1 + \frac{1}{2} \rho V_1^2 + \rho g z_1 = P_2 + \frac{1}{2} \rho V_2^2 + \rho g z_2 + \rho w_s + \Delta P_f$$

$$\text{Utn förluster: } \begin{cases} w_s = 0 \\ \Delta P_f = 0 \end{cases}$$

$$\text{Vet att: } z_1 = z_2$$

$$P_1 = P_a - \rho g h$$

$$P_2 = P_a$$

$$\Rightarrow P_a - \rho g h + \frac{1}{2} \rho V_1^2 = P_a + \frac{1}{2} \rho V_2^2$$

Använd kontinuitet för att uttrycka

V_2 i termer av V_1 :

{ 1-dim, stationär strömning }

$$Q_1 = Q_2 \Leftrightarrow V_1 A_1 = V_2 A_2$$

$$V_2 = \frac{\pi \frac{D_1^2}{4}}{\pi \frac{D_2^2}{4}} \cdot V_1 = \left(\frac{D_1}{D_2} \right)^2 V_1$$

$$\Rightarrow \cancel{P_a} - \rho g h + \frac{1}{2} \rho V_1^2 = \cancel{P_a} + \frac{1}{2} \rho \left(\frac{D_1}{D_2} \right)^4 V_1^2$$

$$\frac{1}{2} \rho \left[1 - \left(\frac{D_1}{D_2} \right)^4 \right] V_1^2 = \rho g h$$

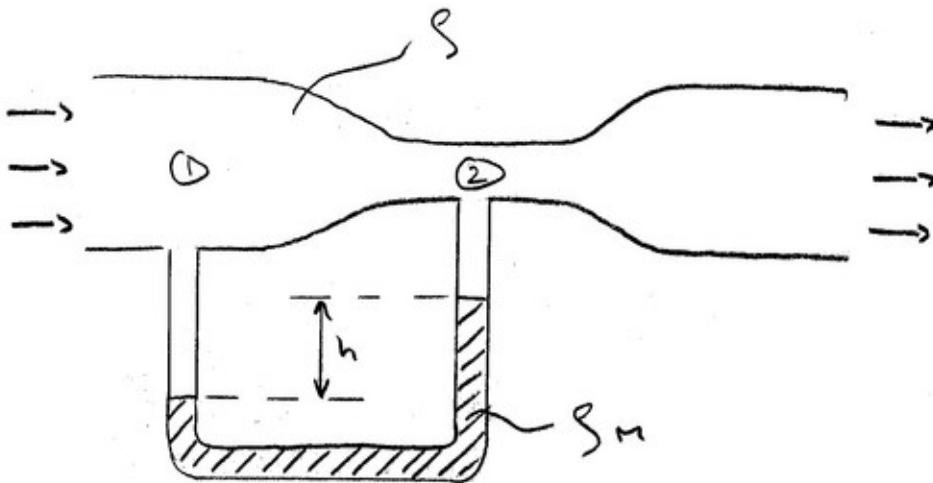
$$V_1 = \sqrt{\frac{2gh}{1 - (D_1/D_2)^4}}$$

3.27

3.18

VENTURIMETER:

$$VISA ATT: Q = \frac{A_2}{\sqrt{1 - (D_2/D_1)^4}} \sqrt{\frac{2gh(\rho_m - \rho)}{\rho}}$$



Bernoullis eku. (stationärt, inkompr, förlustfritt)

$$P_1 + \rho \frac{V_1^2}{2} = P_2 + \rho \frac{V_2^2}{2}$$

$$\text{Hydrostatik ger: } P_1 - P_2 = (\rho_m - \rho)gh$$

$$\text{Kontinuitet ger: } Q_1 = Q_2 = Q \\ \Rightarrow A_1 V_1 = A_2 V_2 \Rightarrow V_1 = \frac{D_2^2}{D_1^2} V_2$$

Bernoulli:

$$\frac{1}{2} \rho (V_1^2 - V_2^2) + P_1 - P_2 = 0$$

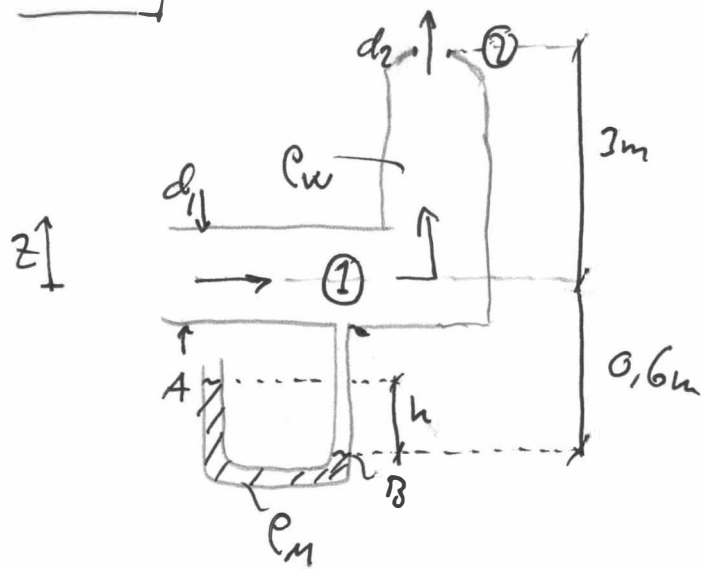
$$\frac{1}{2} \rho \left[\left(\left(\frac{D_2^2}{D_1^2} \frac{Q}{A_2} \right)^2 - \left(\frac{Q}{A_2} \right)^2 \right) + (\rho_m - \rho)gh = 0 \right]$$

$$\left(\frac{Q}{A_2} \right)^2 \left[1 - \left(\frac{D_2}{D_1} \right)^4 \right] = \frac{2gh(\rho_m - \rho)}{\rho}$$

$$Q = \frac{A_2}{\sqrt{1 - (D_2/D_1)^4}} \sqrt{\frac{2gh(\rho_m - \rho)}{\rho}}$$

V. S. V. !

P3.131 / 3.19



known:

$$\begin{aligned} V_1 &= 0.5 \text{ m/s} \\ d_1 &= 0.075 \text{ m} \\ d_2 &= 0.03 \text{ m} \\ \rho_w &= 998 \text{ kg/m}^3 \\ \rho_m &= 13550 \text{ kg/m}^3 \end{aligned} \quad \left. \vphantom{\begin{aligned} V_1 &= 0.5 \text{ m/s} \\ d_1 &= 0.075 \text{ m} \\ d_2 &= 0.03 \text{ m} \\ \rho_w &= 998 \text{ kg/m}^3 \\ \rho_m &= 13550 \text{ kg/m}^3 \end{aligned}} \right\} \text{Table A3}$$

Task: Calculate h

Assume: No losses

Solution: Use B.E (3.54) for frictionless flow between ① & ②
* Use hydrostatic pressure through manometer (2.14)

Find pressure at ①: (3.54)

$$P_1 + \rho_w \frac{V_1^2}{2} + \rho_w g z_1 = P_2 + \rho_w \frac{V_2^2}{2} + \rho_w g z_2 \Rightarrow$$

$$P_1 - P_2 = \frac{\rho_w}{2} (V_2^2 - V_1^2) + \rho_w g (z_2 - z_1) \quad [A]$$

Continuity through the pipe gives V_2 since $Q_2 \equiv Q_1 \Rightarrow V_2 A_2 = V_1 A_1 \Rightarrow$
 $V_2 = V_1 \frac{A_1}{A_2} = V_1 \frac{d_1^2}{d_2^2} \quad (A = \frac{\pi d^2}{4}) \quad \text{this in [A]}$

$$P_1 - P_2 = \frac{\rho_w V_1^2}{2} \left[\left(\frac{d_1}{d_2} \right)^4 - 1 \right] + \rho_w g (\overbrace{z_2 - z_1}^{3\text{m}}) = 34.11 \text{ kPa}$$

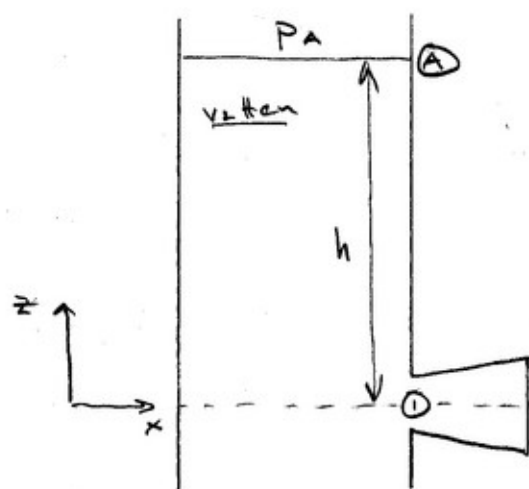
Hydrostatic pressure through the manometer ① $\rightarrow$ ④, and use the fact that $P_A = P_2$ (Atmospheric pressure)

$$\begin{aligned} (2.14) \quad \frac{P_1 - P_2}{\rho_m g} &= (P_1 - P_B) + (P_B - P_A) = -\rho_w g (z_1 - z_B) - \rho_m g (\overbrace{z_B - z_A}^{-h}) \Rightarrow \\ h &= \frac{(P_1 - P_2) + \rho_w g (z_1 - z_B)}{\rho_m g} = 0.30 \text{ m} \end{aligned}$$

We should ideally place ① at the lower well when going through manometer
 i.e. $\rightarrow$ $z_1 - z_B = 0.6 - 0.075/2 \Rightarrow h = \frac{(P_1 - P_2) + \rho_w g (0.6 - 0.075/2 - 0)}{\rho_m g} = 0.298 \text{ m}$
 (Very small diff to 0.30)

3.24

3.20



x Försumma förluster

x Sök h sådant att

 $p_1 = p_v$ (ämbildnings
tryck för
vatten)x $p_v = 4.242 \text{ kPa}$ vid $T = 30^\circ\text{C}$
(tabell A.5 s 812)

x fri stråle

x $D_2 = 5 \text{ cm}$ x $D_1 = 8 \text{ cm}$

Bernoullis ekv. med inlopp ④ och utlopp ①

$$p_A + \rho g h = p_v + \frac{1}{2} \rho v_1^2 \quad (1)$$

Bernoullis ekv. ① → ② ($z_1 = z_2$)

$$p_v + \frac{1}{2} \rho v_1^2 = p_A + \frac{1}{2} \rho v_2^2 \quad (2)$$

Kontinuitets ekv. mellan ① och ②

$$(\text{från 3.21}) : v_2 = \left(\frac{D_1}{D_2}\right)^2 v_1 \quad (3)$$

Sätt in (3) i (2) och lös för $\frac{1}{2} \rho v_1^2$:

$$p_v + \frac{1}{2} \rho v_1^2 = p_A + \frac{1}{2} \rho \left(\frac{D_1}{D_2}\right)^4 v_1^2$$

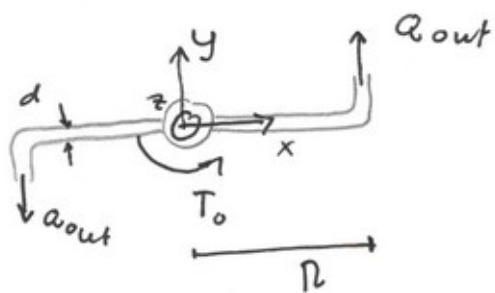
$$\Rightarrow \frac{1}{2} \rho v_1^2 = \frac{p_A - p_v}{1 - \left(\frac{D_1}{D_2}\right)^4}$$

Sätt in i (1) och lös för h:

$$p_A + \rho g h = p_v + \frac{p_A - p_v}{1 - \left(\frac{D_1}{D_2}\right)^4}$$

$$\rho g h = (p_A - p_v) \left[\frac{1}{1 - \left(\frac{D_1}{D_2}\right)^4} - 1 \right] = \frac{p_A - p_v}{\left(\frac{D_2}{D_1}\right)^4 - 1}$$

$$h = \frac{p_A - p_v}{\rho g \left[\left(\frac{D_2}{D_1}\right)^4 - 1 \right]} \approx \underline{\underline{1.76 \text{ m}}}$$



known:

$$\dot{Q}_{in} = \frac{15}{1000 \cdot 60} \frac{m^3}{s}$$

$$R = 0.15 m$$

$$d = 0.006 m$$

$$\rho = 998 kg/m^3$$

Task: a) Find T_0 so that the sprinkler is fixed (not moving)

b) find the runaway speed (rotation if no retarding torque)

Assume: steady-state, incomp., fix CV (a)

Solution: Use R.T.T. for conservation of angular momentum (3.59)

$$\begin{aligned} \sum \tilde{M}_O &= \frac{d}{dt} \left(\int_{CV} (\tilde{r} \times \tilde{v}) \rho dV \right) + \int_{CS} (\tilde{r} \times \tilde{v}) \rho (\tilde{v} \cdot \tilde{n}) dA = \\ &= \int_{in} (\tilde{r} \times \tilde{v})_{in} \rho (\tilde{v} \cdot \tilde{n})_{in} dA + 2 \int_{out} (\tilde{r} \times \tilde{v})_{out} \rho (\tilde{v} \cdot \tilde{n})_{out} dA = \\ &= 2 \int_{out} \left(\begin{bmatrix} R \\ 0 \\ 0 \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ v_{out} \end{bmatrix} \right) \rho \left(\begin{bmatrix} 0 \\ v_{out} \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right) dA = \dots = 2 \int_{out} \rho v_{out}^2 R dA \\ &= 2 \rho v_{out}^2 R \cdot A \end{aligned}$$

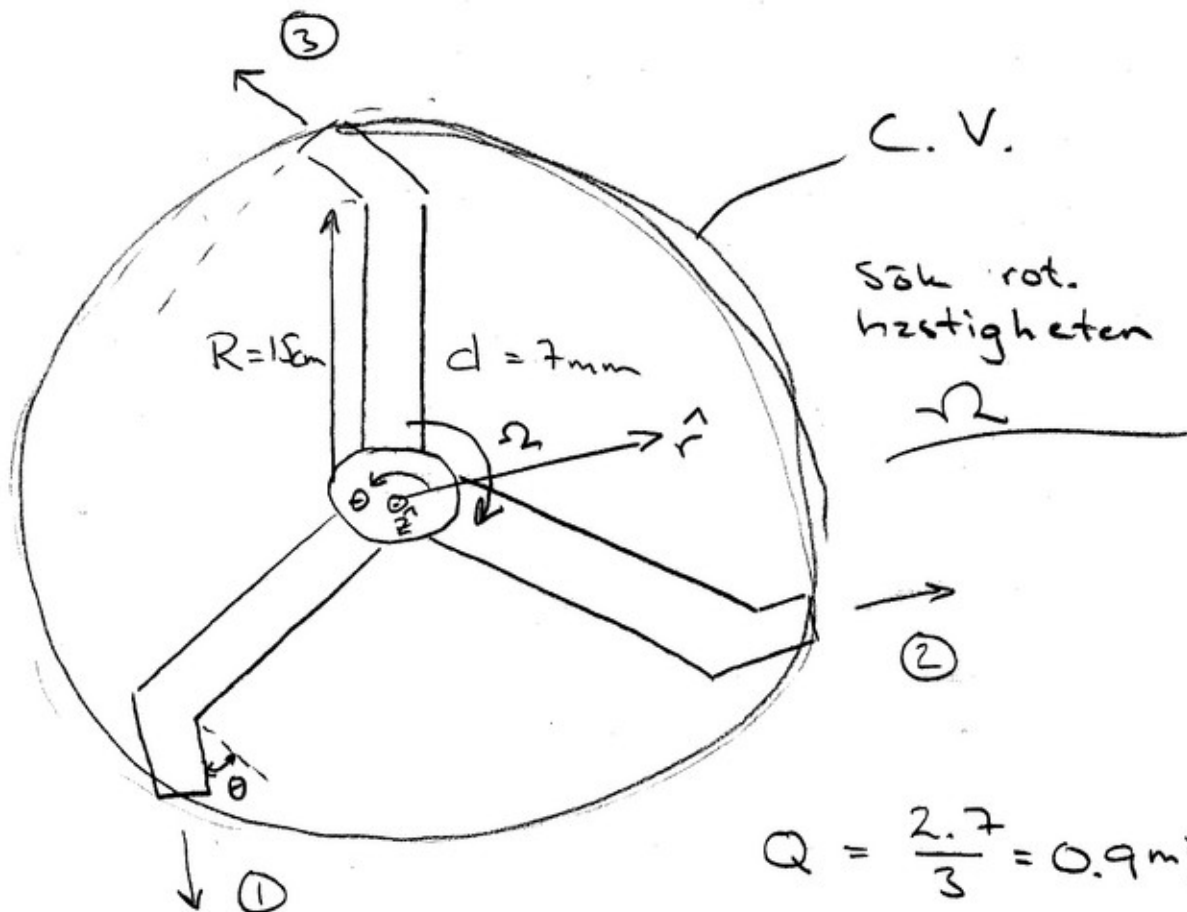
$$v_{out} = \frac{\dot{Q}_{in}}{2A} = \frac{2 \dot{Q}_{in}}{\pi d^2} \Rightarrow \sum M_O = T_0 = 2 \rho \left(\frac{2 \dot{Q}_{in}}{\pi d^2} \right)^2 R \cdot \frac{\pi d^2}{4} = 0.165 Nm$$

b) Rotational speed if $T_0 = 0$, i.e. no braking torque.

$$\omega = \frac{v}{R} = \frac{2 \dot{Q}_{in}}{\pi d^2 R} = 29.47 \text{ rad/s} = 29.47 \cdot \frac{60}{2\pi} \text{ rpm} = 281 \text{ rpm}$$

3.19

3.22 (1)

Sök rot.
hastigheten Ω

$$Q = \frac{2.7}{3} = 0.9 \text{ m}^3/\text{h}$$

$$= 2.5 \cdot 10^{-4} \text{ m}^3/\text{s}$$

Impulsmomentsatsen:

$$\text{RTT med } \begin{cases} \vec{B} = \vec{H}_0 = \int \vec{r} \times \vec{v} \, dm \\ \vec{B} = \frac{d\vec{H}_0}{dt} = \vec{r} \times \vec{v} \end{cases}$$

$$\frac{d\vec{H}_0}{dt} = \sum \vec{M}_0 = \frac{d}{dt} \int_{\text{C.V.}} (\vec{r} \times \vec{v}) \rho \, dV + \int_{\text{C.S.}} (\vec{r} \times \vec{v}) \rho (\vec{v} \cdot \hat{n}) \, dA$$

gäller för icke-deformerbar C.V.

$$\frac{d}{dt} \int_{\text{C.V.}} (\vec{r} \times \vec{v}) \rho \, dV = 0 \quad \text{ty stationärt och inkompressibelt}$$

$$\sum \vec{M}_0 = \vec{0} \quad (\text{inga yttre krafter})$$

$$\Rightarrow \int_{\text{①}} (\vec{r} \times \vec{v}) \rho (\vec{v} \cdot \hat{n}) \, dA + \int_{\text{②}} (\vec{r} \times \vec{v}) \rho (\vec{v} \cdot \hat{n}) \, dA + \int_{\text{③}} \dots = 0$$

3.19

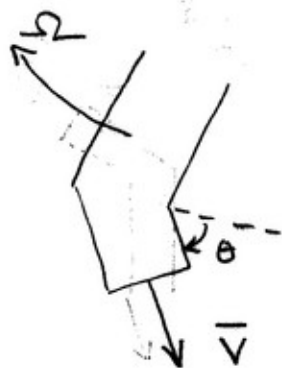
forts.

Beräkna integral över utlopp ①, ② och ③

3.22 (2)

$$\hat{n} = \sin \theta \hat{r} + \cos \theta \hat{\theta}$$

$$\vec{V} = \frac{Q}{A} \sin \theta \hat{r} + \left(\frac{Q}{A} \cos \theta - \Omega R \right) \hat{\theta}$$



$$\vec{r} \times \vec{V} = R \left(\frac{Q}{A} \cos \theta - \Omega R \right) \hat{z}$$

$$\int (\vec{r} \times \vec{V}) \cdot \hat{n} dA = (\vec{r} \times \vec{V}) (\dot{m}_1 + \dot{m}_2 + \dot{m}_3) = 0$$

$$\Rightarrow \vec{r} \times \vec{V} = 0$$

$$R \left(\frac{Q}{A} \cos \theta - \Omega R \right) = 0$$

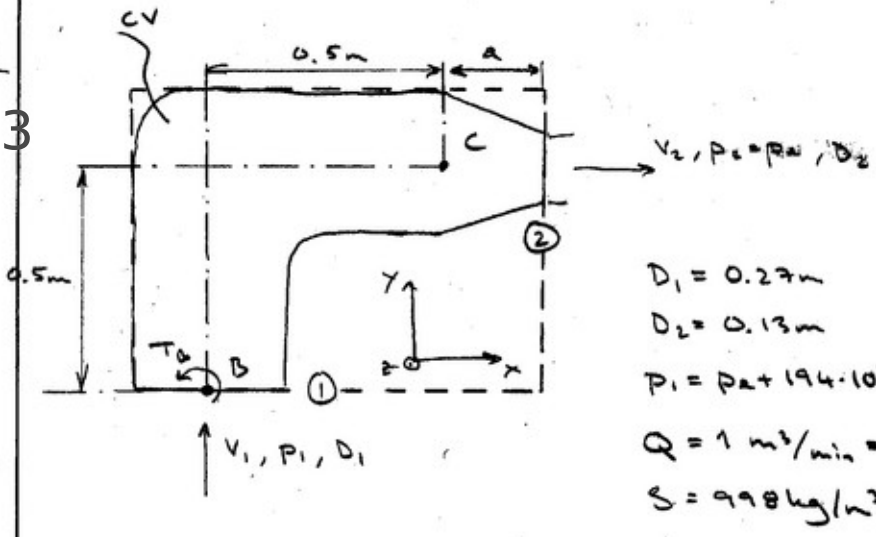
$$\Omega = \frac{Q \cos \theta}{A \cdot R} = \frac{2.5 \cdot 10^{-4} \cos \theta}{\pi \cdot 0.0035^2 \cdot 0.15}$$

$$a) \Omega \approx 43.3 \text{ rad/s} \approx \underline{\underline{414 \text{ varv/min}}}$$

$$b) \Omega \approx 33.2 \text{ rad/s} \approx \underline{\underline{317 \text{ varv/min}}}$$

3.23

3.23



$D_1 = 0.27 \text{ m}$
 $D_2 = 0.13 \text{ m}$
 $P_1 = P_2 = 194 \cdot 10^3 \text{ Pa}$
 $Q = 1 \text{ m}^3/\text{min} = 0.0167 \text{ m}^3/\text{s}$
 $\rho = 998 \text{ kg/m}^3$

Impulsmoment satsen:

RTT med $B = \bar{H}_0 = \int_{CV} (\bar{r} \times \bar{V}) dm$

$$B = \frac{d\bar{H}_0}{dt} = \bar{r} \times \bar{V}$$

$$\sum \bar{M}_B = \frac{\partial}{\partial t} \int_{CV} (\bar{r} \times \bar{V}) \rho dV + \int_{CS} (\bar{r} \times \bar{V}) \rho (\bar{V} \cdot \hat{n}) dA$$

Fix C.V. och stationært: $\frac{\partial}{\partial t} = 0$

$$\sum \bar{M}_B = \sum (\bar{r} \times \bar{F}_{ext}) + T_B \hat{z} = \int_{CS} (\bar{r} \times \bar{V}) \rho (\bar{V} \cdot \hat{n}) dA =$$

{ räknar bara med övertryck och $\bar{F}_1 = \bar{0}$ }

$$= \int_{CS} (\bar{0} \times \bar{V}_1) \rho (\bar{V}_1 \cdot (-\hat{n})) dA + \int_{CS} (\bar{r}_2 \times \bar{V}_2) \rho (\bar{V}_2 \cdot \hat{n}) dA =$$

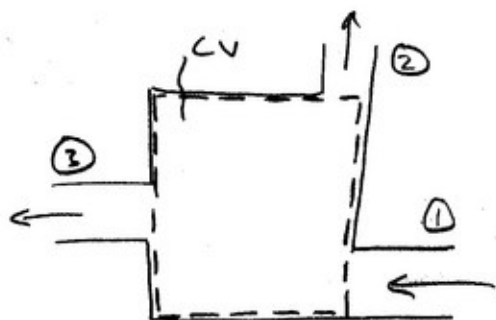
$$= \left\{ \begin{aligned} \bar{r}_2 &= (0.5 + z, 0.5, 0), \bar{V}_2 = (V_2, 0, 0) \\ \bar{r}_2 \times \bar{V}_2 &= \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0.5 + z & 0.5 & 0 \\ V_2 & 0 & 0 \end{vmatrix} = -0.5 V_2 \hat{z} \\ \bar{V}_2 \cdot \hat{n} &= V_2 \end{aligned} \right\} = \int -0.5 V_2 \hat{z} \rho V_2 dA$$

$$-0.5 \rho V_2^2 A_2 \hat{z} = -\frac{2 \rho Q^2}{\pi D_2^3} \hat{z} \approx 10.5 \text{ Nm medels}$$

3.20

3.24

$$\dot{Q} - \dot{W}_s = \frac{\partial}{\partial t} \int_{CV} \left(\hat{u} + \frac{1}{2} V^2 + g z \right) \rho dV + \int_{CS} \left(\hat{u} + \frac{1}{2} V^2 + g z \right) \rho (\vec{V} \cdot \hat{n}) dA$$



$$V_1 = \frac{4Q_1}{\pi D_1^2} = 9.60 \text{ m/s}$$

$$V_2 = \frac{4Q_2}{\pi D_2^2} = 7.22 \text{ m/s}$$

$$V_3 = \frac{4Q_3}{\pi D_3^2} = 26.50 \text{ m/s}$$

$$\rho = 998 \text{ kg/m}^3$$

$$\begin{cases} D_1 = 0.09 \text{ m} \\ Q_1 = 0.0611 \text{ m}^3/\text{s} \\ P_1 = 150 \cdot 10^3 \text{ Pa} \end{cases}$$

$$\begin{cases} D_2 = 0.07 \text{ m} \\ Q_2 = 0.0278 \text{ m}^3/\text{s} \\ P_2 = 225 \cdot 10^3 \text{ Pa} \end{cases}$$

$$\begin{cases} D_3 = 0.04 \text{ m} \\ Q_3 = 0.0333 \text{ m}^3/\text{s} \\ P_3 = 265 \cdot 10^3 \text{ Pa} \end{cases}$$

$$\text{Stationært: } \frac{\partial}{\partial t} = 0$$

$$\text{Försumma värmeöverföring: } \dot{Q} = 0$$

$$\text{Försumma gravitation: } g z = 0$$

$$\text{Försumma temp. effekter: } \hat{u} = \text{const.}$$

$$-\dot{W}_s = \int_{CS} \left(\hat{u} + \frac{P}{\rho} + \frac{1}{2} V^2 \right) \rho (\vec{V} \cdot \hat{n}) dA$$

1-dim in- och utlopp:

$$-\dot{W}_s = \rho Q_2 \left(\hat{u} + \frac{P_2}{\rho} + \frac{1}{2} V_2^2 \right) + \rho Q_3 \left(\hat{u} + \frac{P_3}{\rho} + \frac{1}{2} V_3^2 \right)$$

$$- \rho Q_1 \left(\hat{u} + \frac{P_1}{\rho} + \frac{1}{2} V_1^2 \right) =$$

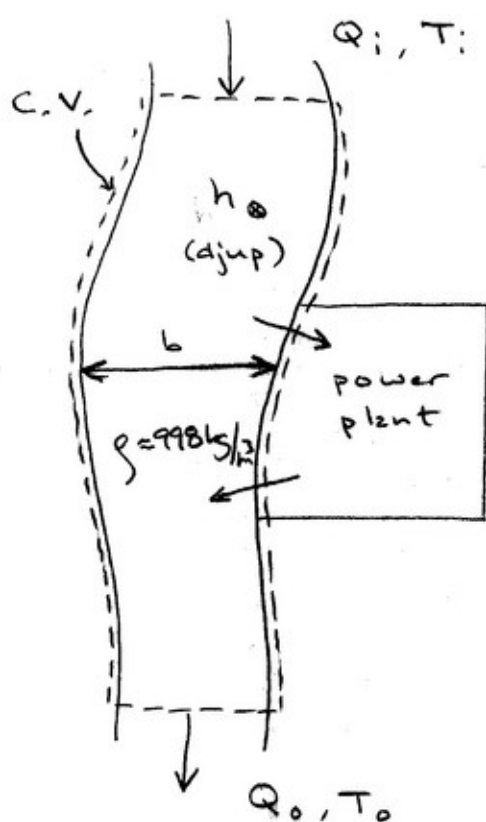
$$= Q_2 \left(P_2 + \frac{1}{2} \rho V_2^2 \right) + Q_3 \left(P_3 + \frac{1}{2} \rho V_3^2 \right) - Q_1 \left(P_1 + \frac{1}{2} \rho V_1^2 \right)$$

$$= 15497 \text{ W}$$

$$\underline{\underline{\dot{W}_s = -15.5 \text{ kW}}}$$

3.26

3.25

KÄNT

$$Q_i = 2.5 \text{ m}^3/\text{s}$$

$$T_i = 18^\circ\text{C}$$

$$b = 45 \text{ m}$$

$$h = 2.7 \text{ m}$$

$$d\hat{h} = c_p dT$$

$$\dot{Q} = 55 \text{ MW}$$

$$c_p = 4182 \text{ J/kgK}$$

Kontinuitet ger: $Q_i = Q_o \Rightarrow \dot{m} = \dot{m}_i = \dot{m}_o$

Använd nu energi ekvationen:

$$\text{RTT för } \begin{cases} B = E \\ B = \frac{dE}{dm} = e \end{cases}$$

$$(3.63) \quad \underbrace{\dot{Q} - \dot{W}_s - \dot{W}_v}_{=0} = \underbrace{\frac{\partial}{\partial t} \int_{\text{C.V.}} \left(\hat{u} + \frac{1}{2} v^2 + gz \right) \rho dV}_{=0 \text{ ty stationärt}}$$

$$+ \int_{\text{C.S.}} \left(\hat{h} + \frac{1}{2} v^2 + gz \right) \rho (\vec{V} \cdot \hat{n}) dA$$

(3.64) 1-dim. in- och utlopp:

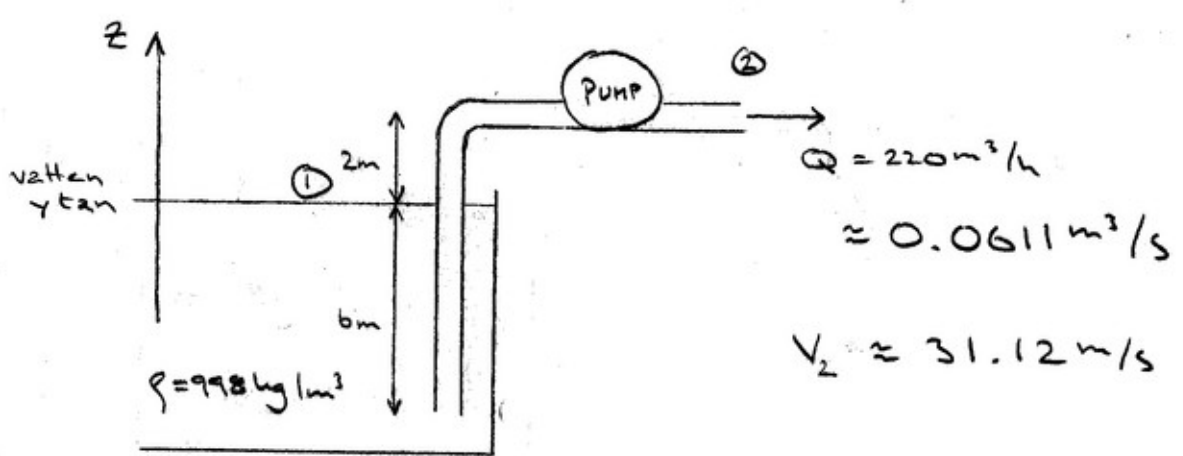
$$\dot{Q} = \left(\hat{h} + \frac{1}{2} v^2 + gz \right)_o \dot{m}_o - \left(\hat{h} + \frac{1}{2} v^2 + gz \right)_i \dot{m}_i$$

$$\dot{Q} = \dot{m} (\hat{h}_o - \hat{h}_i) = \dot{m} c_p (T_o - T_i)$$

$$T_o = \frac{\dot{Q}}{\dot{m} c_p} + T_i = \frac{55 \cdot 10^6}{998 \cdot 2.5 \cdot 4182} + 18 \approx \underline{\underline{23.3^\circ\text{C}}}$$

3.25

3.26



$$h_f = 5 \text{ m} \Rightarrow \Delta p_f = \rho g h_f = 48951.9 \text{ Pa}$$

ANVÄND BERNOLLI'S UTVIKLADE EKV.

$$\cancel{p_1} + \underbrace{\frac{\rho V_1^2}{2}}_{=0} + \rho g z_1 = \cancel{p_2} + \frac{\rho V_2^2}{2} + \rho g z_2 + \Delta p_f - \rho w_{\text{pump}}$$

(-) since a pump adds energy

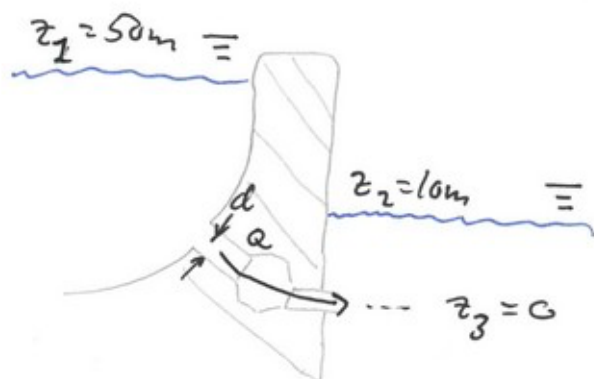
$$\rho w_{\text{pump}} = \frac{\rho V_2^2}{2} + \rho g (z_2 - z_1) + \Delta p_f$$

$$P_{\text{pump}} = Q \rho w_{\text{pump}} = 0.0611 \left(\frac{998 \cdot 31.12^2}{2} + 998 \cdot 9.81 \cdot 2 + 48951.9 \right)$$

$$\approx \underline{\underline{33.7 \text{ kW}}}$$

P 3.184 / 3.27 (1)

known:



$$\begin{aligned} P &= 25 \text{ MW} & d &= 4 \text{ m} \\ z_1 &= 50 \text{ m} \\ z_2 &= 10 \text{ m} & \rho &= 998 \text{ kg/m}^3 \text{ (water)} \\ z_3 &= 0 \text{ m} \\ h_f &= \frac{3.5 V^2}{2g} \end{aligned}$$

Task: Find the flow rate through the turbine (two possibilities).
* Which flow rate has a better 'conversion efficiency'?

Assume: Steady-state, incompressible flow

Solution: Use the definition of pump/turbine power
* combine this with the energy equation (7.75)

The turbine power: $P = \rho g h_{\text{turb}} Q \Rightarrow h_{\text{turb}} = \frac{P}{\rho g Q}$ [A]

Use the energy equation between 1 and 2. (7.75)

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_{\text{turb}} - \cancel{h_{\text{pump}}} + h_f \Rightarrow$$

$P_1 = P_2$ $V_1 = V_2$ $P_1 = P_2$ $V_1 = V_2$ h_{pump} (no pump)

$$z_1 = z_2 + h_{\text{turb}} + h_f \Rightarrow 0 = \frac{P}{\rho g Q} + \frac{3.5 V^2}{2g} - \underbrace{\Delta z}_{z_1 - z_2}$$

[A]

Use the relation between velocity & flow rate, i.e. $Q = V \cdot A \Rightarrow V = \frac{Q}{A} = \frac{4Q}{\pi d^2} \Rightarrow$

$$0 = \frac{P}{\rho g Q} + \frac{3.5 \left(\frac{4Q}{\pi d^2} \right)^2}{2g} - \Delta z \Rightarrow \frac{P}{\rho g Q} + \frac{28 Q^2}{g \pi^2 d^4} - \Delta z = 0$$

Multiply by Q : $\frac{P}{\rho g} + \frac{28 Q^3}{g \pi^2 d^4} - \Delta z Q = 0$

Multiply by $\frac{g \pi^2 d^4}{28}$: $\underbrace{\frac{P}{\rho g} \frac{g \pi^2 d^4}{28}}_A + Q^3 - \underbrace{\Delta z \frac{g \pi^2 d^4}{28}}_B Q = 0 \Rightarrow$

3.27 (2)

$$A = 2,260 \dots \cdot 10^6, \quad B = 3.540 \dots \cdot 10^3$$

We have: $Q^3 - BQ + A = 0 \quad [B]$

This equation needs to be solved iteratively (or numerically)

Start by guessing Q to satisfy the $[B]$ equation

Initial guess $Q = 50 \text{ m}^3/\text{s}$ in $[B] = 6.15e6$

New guess $Q = 60 \text{ m}^3/\text{s}$ in $[B] = 3.52e5$

— " — $Q = 70$ — " — $[B] = 1.25e5$

— " — $Q = 80$ — " — $[B] = -6.03e4$ } Root must be between 70 & 80 m^3/s

— " — $Q = 75$ — " — $[B] = 2.66e4$

— " — $Q = 76$ — " — $[B] = 8.34e3$

By continue guessing/iterate the three roots are found

$$Q_1 = 76.46 \text{ m}^3/\text{s}$$

$$Q_2 = 137.90 \text{ m}^3/\text{s}$$

$$Q_3 = -214.37 \text{ m}^3/\text{s} \leftarrow \text{Impossible!!!}$$

The Q_1 ($76.46 \text{ m}^3/\text{s}$) must have a better "Conversion efficiency"

Since the head loss term, h_f , will be smaller with a smaller flow rate.

$$h_f(Q_1) = 6.6 \text{ m}$$

$$h_f(Q_2) = 21.5 \text{ m} \leftarrow \text{larger energy loss in for the turbine.}$$