

Fluid Mechanics - MTF053

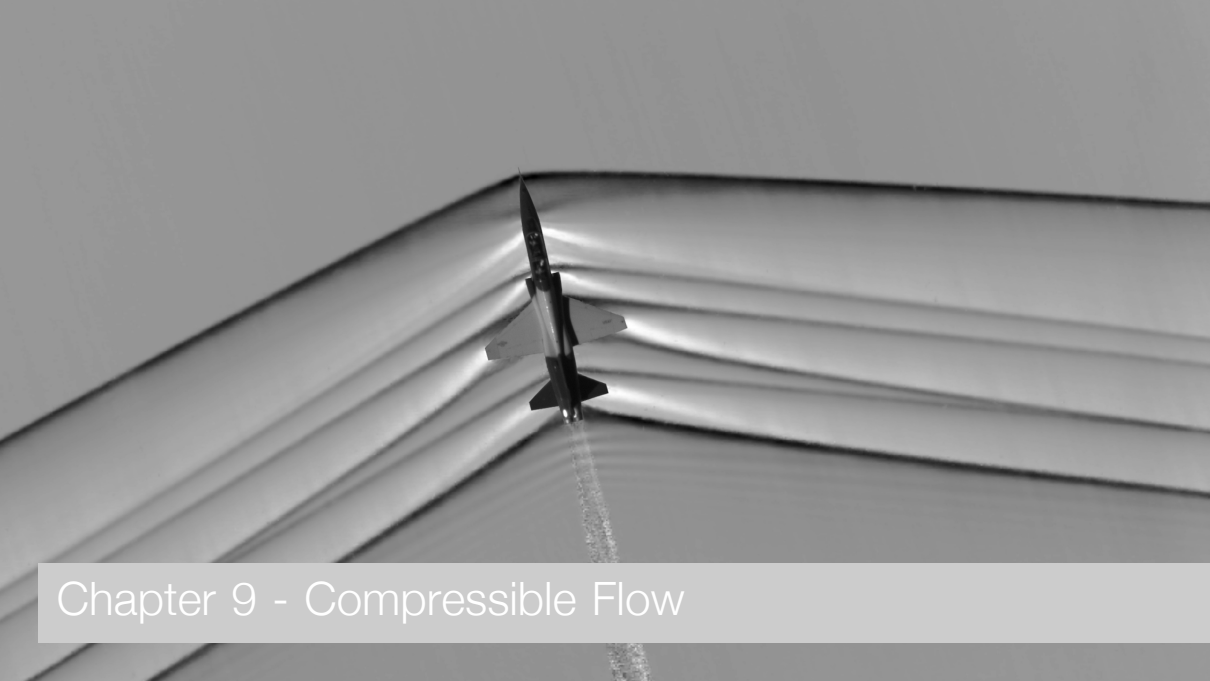
Lecture 19

Niklas Andersson

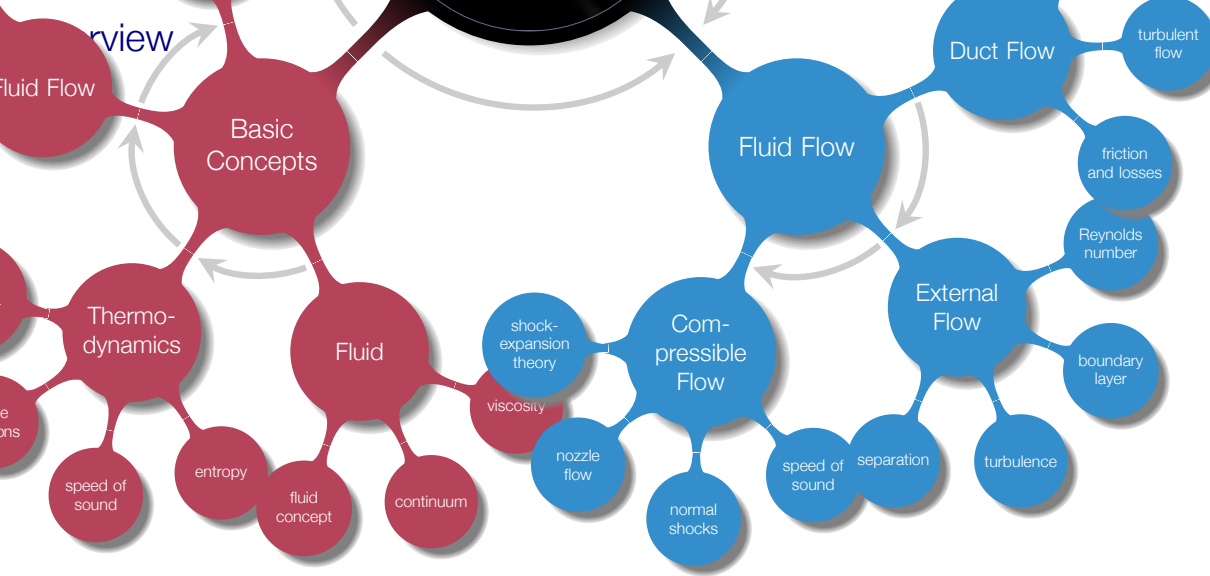
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Chapter 9 - Compressible Flow

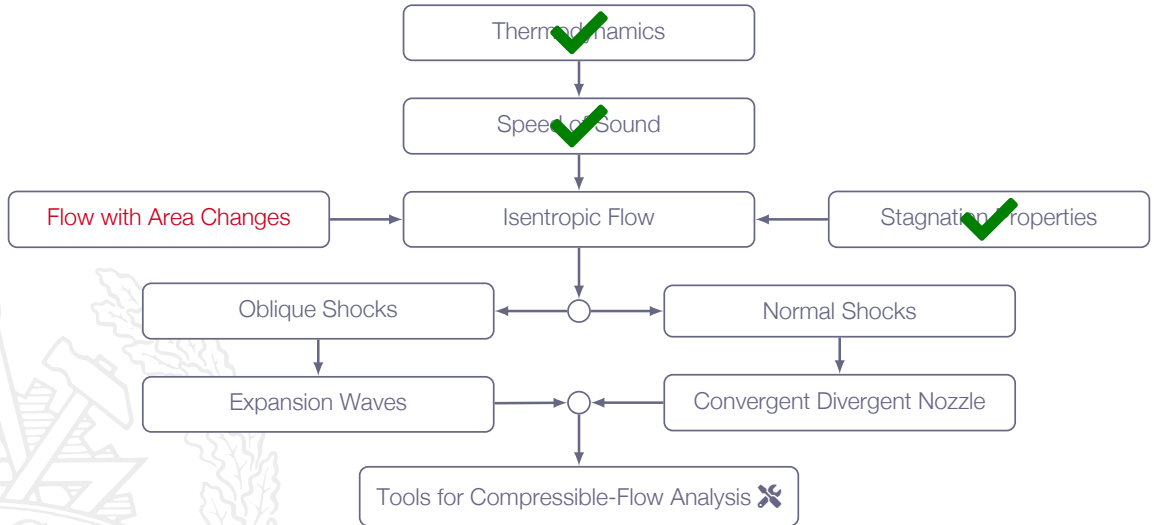


Learning Outcomes

- 4 Be **able to categorize** a flow and **have knowledge about** how to select applicable methods for the analysis of a specific flow based on category
- 37 **Understand and explain** basic concepts of compressible flows (the gas law, speed of sound, Mach number, isentropic flow with changing area, normal shocks, oblique shocks, Prandtl-Meyer expansion)

Let's go supersonic ...

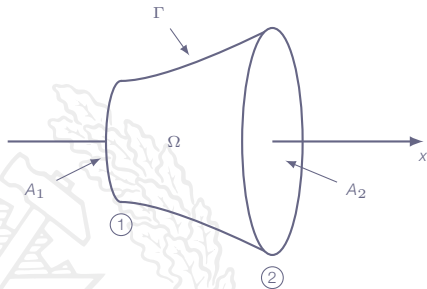
Roadmap - Compressible Flow



Isentropic Quasi-1D Flow

Quasi-1D Assumptions:

1. Flow properties varies in one direction only (x)
2. The flow area is a smooth function $A = A(x)$
3. Steady-state, inviscid and isentropic flow



The Area-Velocity Relation

Continuity:

$$\rho(x)V(x)A(x) = \text{const} \Rightarrow d(\rho VA) = 0 \Rightarrow AVd\rho + \rho AdV + \rho VdA = 0$$

divide by ρVA gives

$$\frac{d\rho}{\rho} + \frac{dV}{V} + \frac{dA}{A} = 0$$



The Area-Velocity Relation

In the following, isentropic flow is assumed

Stagnation enthalpy:

$$h_o = h + \frac{1}{2}V^2 = \text{const} \Rightarrow dh + VdV = 0$$

The first and second law of thermodynamics:

$$Tds = 0 = dh - \frac{dp}{\rho} \Rightarrow dh = \frac{dp}{\rho}$$

and thus

$$\frac{dp}{\rho} + VdV = 0$$

The Area-Velocity Relation

$$\frac{dp}{\rho} + VdV = 0$$

From the definition of the **speed of sound**

$$dp = a^2 d\rho \Rightarrow a^2 \frac{d\rho}{\rho} + VdV = 0 \Rightarrow \frac{d\rho}{\rho} = -\frac{1}{a^2} VdV$$



The Area-Velocity Relation

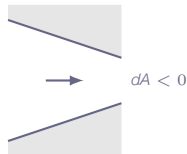
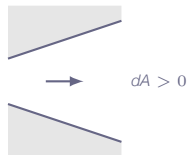
$$\frac{d\rho}{\rho} + \frac{dV}{V} + \frac{dA}{A} = \frac{dV}{V} - \frac{1}{a^2} V dV + \frac{dA}{A} = 0$$

$$\frac{dV}{V} \left(\frac{V^2}{a^2} - 1 \right) = \frac{dA}{A}$$

$$\frac{dV}{V} = \frac{1}{M^2 - 1} \frac{dA}{A} = - \frac{dp}{\rho V^2}$$

The Area-Velocity Relation

$$\frac{dV}{V}(M^2 - 1) = \frac{dA}{A}$$



Subsonic $M < 1$ **Supersonic** $M > 1$

subsonic diffuser

$$dV < 0$$

$$dp > 0$$

supersonic nozzle

$$dV > 0$$

$$dp < 0$$

subsonic nozzle

$$dV > 0$$

$$dp < 0$$

supersonic diffuser

$$dV < 0$$

$$dp > 0$$

The Area-Velocity Relation

$$\frac{dV}{V}(M^2 - 1) = \frac{dA}{A}$$

What happens when $M = 1$?



The Area-Velocity Relation

$$\frac{dV}{V}(M^2 - 1) = \frac{dA}{A}$$

What happens when $M = 1$?

$M = 1$ when $dA = 0$



The Area-Velocity Relation

$$\frac{dV}{V}(M^2 - 1) = \frac{dA}{A}$$

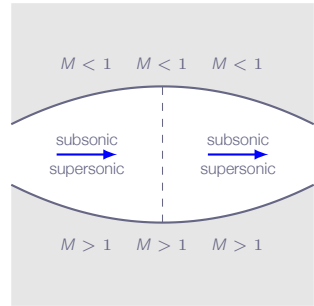
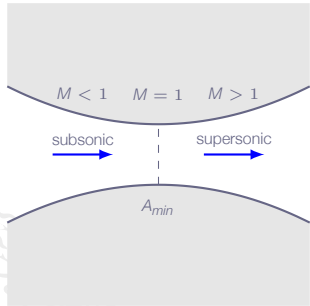
What happens when $M = 1$?

$M = 1$ when $dA = 0$

maximum or minimum area



The Area-Velocity Relation



The Area-Mach-Number Relation

$$\rho AV = \rho^* A^* V^* \Rightarrow \frac{A}{A^*} = \frac{\rho^*}{\rho} \frac{V^*}{V}$$

$$\frac{\rho^*}{\rho} = \frac{\rho^*}{\rho_o} \frac{\rho_o}{\rho} = \left[\frac{2}{\gamma + 1} \left(1 + \frac{\gamma - 1}{2} M^2 \right) \right]^{1/(\gamma - 1)}$$

$$\frac{V^*}{V} = \frac{(\gamma R T^*)^{1/2}}{V} = \frac{(\gamma R T)^{1/2}}{V} \left(\frac{T^*}{T_o} \right)^{1/2} \left(\frac{T_o}{T} \right)^{1/2} = \frac{1}{M} \left[\frac{2}{\gamma + 1} \left(1 + \frac{\gamma - 1}{2} M^2 \right) \right]^{1/2}$$

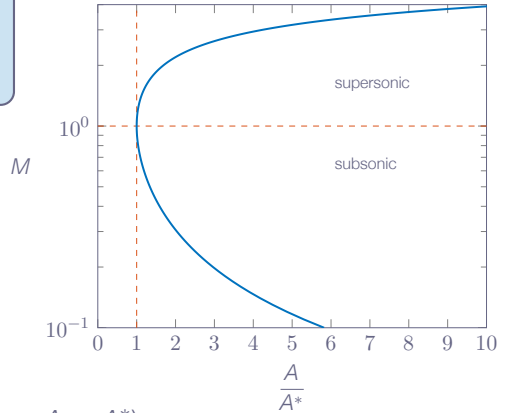
$$\left(\frac{A}{A^*} \right)^2 = \frac{1}{M^2} \left[\frac{2 + (\gamma - 1) M^2}{\gamma + 1} \right]^{(\gamma + 1)/(\gamma - 1)}$$

The Area-Mach-Number Relation

$$\left(\frac{A}{A^*}\right)^2 = \frac{1}{M^2} \left[\frac{2 + (\gamma - 1)M^2}{\gamma + 1} \right]^{(\gamma+1)/(\gamma-1)}$$

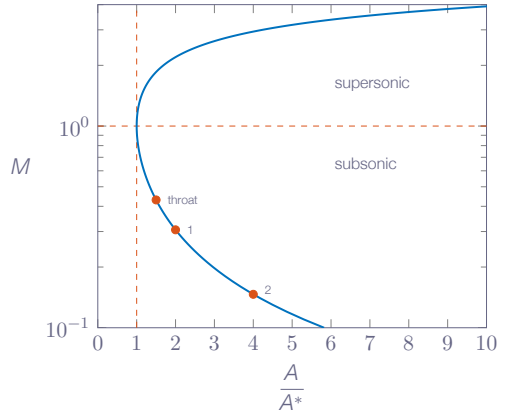
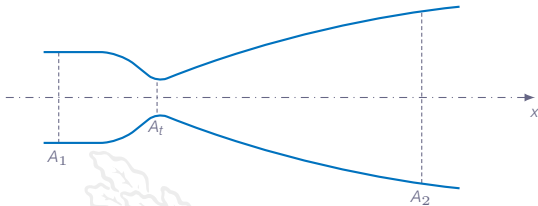
Note!

Two possible solutions for each value of $\frac{A}{A^*}$:
one subsonic and one supersonic (except when $A = A^*$)



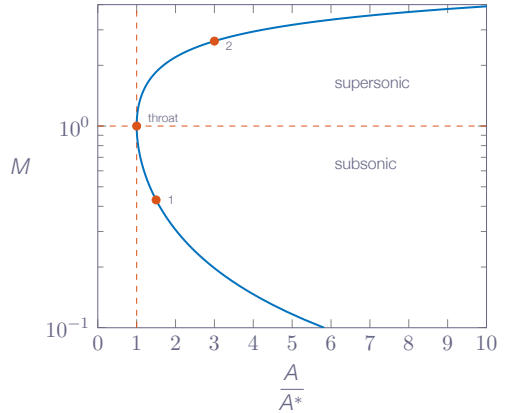
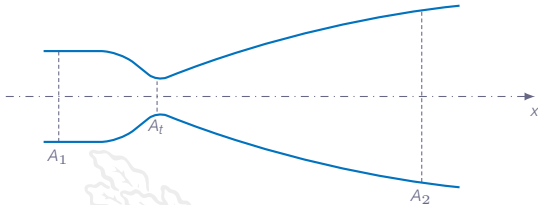
The Area-Mach-Number Relation

Sub-critical (**non-choked**) nozzle flow



The Area-Mach-Number Relation

Critical (**choked**) nozzle flow



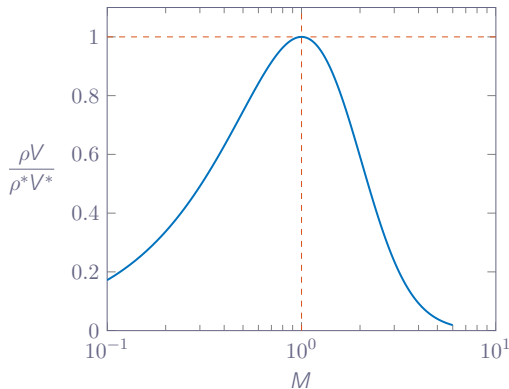
Choking

$$\rho V A = \rho^* V^* A^* \Rightarrow \frac{A^*}{A} = \frac{\rho V}{\rho^* V^*}$$

ρV – massflow per unit area

From the area-Mach-number relation:

$$\frac{A^*}{A} = \begin{cases} < 1 & \text{if } M < 1 \\ 1 & \text{if } M = 1 \\ < 1 & \text{if } M > 1 \end{cases}$$



The **maximum** possible **mass flow** through a duct is achieved when its throat reaches **sonic conditions**

Choking

$$1. \dot{m}_{max} = \rho^* A^* V^*$$

$$2. \frac{\rho^*}{\rho_o} = \left(\frac{2}{\gamma + 1} \right)^{1/(\gamma-1)}$$

$$3. V^* = \sqrt{\gamma R T^*}$$

$$4. \frac{T^*}{T_o} = \frac{2}{\gamma + 1}$$

$$\dot{m}_{max} = \frac{A^* p_o}{\sqrt{T_o}} \sqrt{\frac{\gamma}{R} \left(\frac{2}{\gamma + 1} \right)^{\frac{(\gamma+1)}{(\gamma-1)}}}$$