

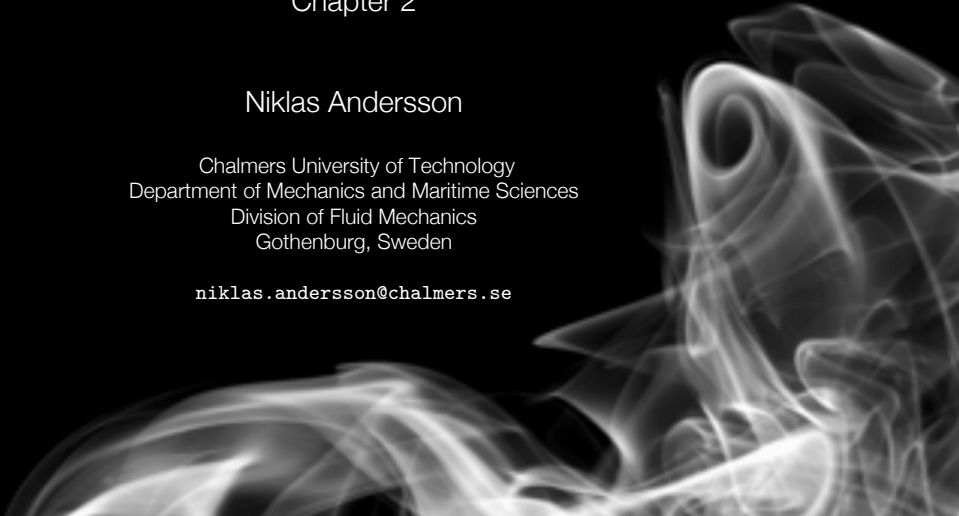
Fluid Mechanics - MTF053

Chapter 2

Niklas Andersson

Chalmers University of Technology
Department of Mechanics and Maritime Sciences
Division of Fluid Mechanics
Gothenburg, Sweden

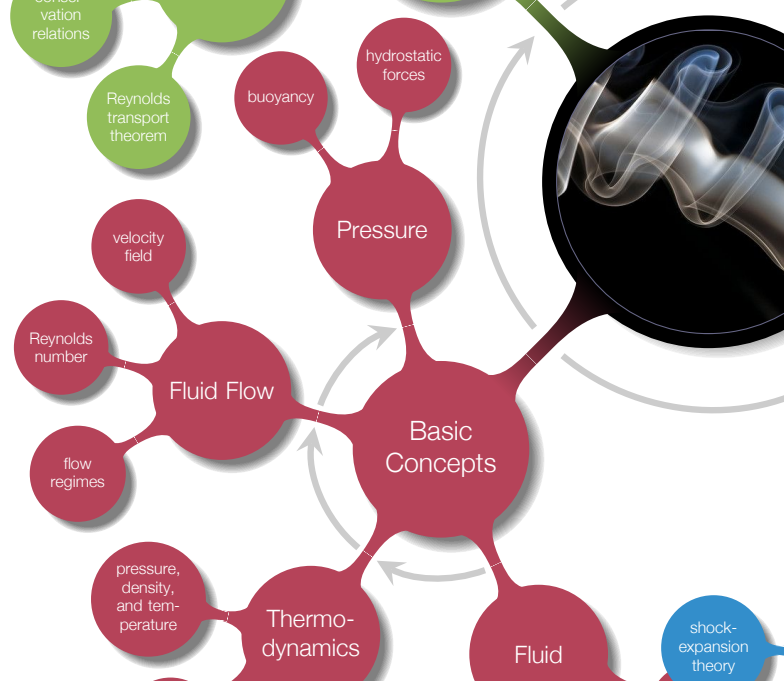
`niklas.andersson@chalmers.se`





Chapter 2 - Pressure Distribution in a Fluid

Overview



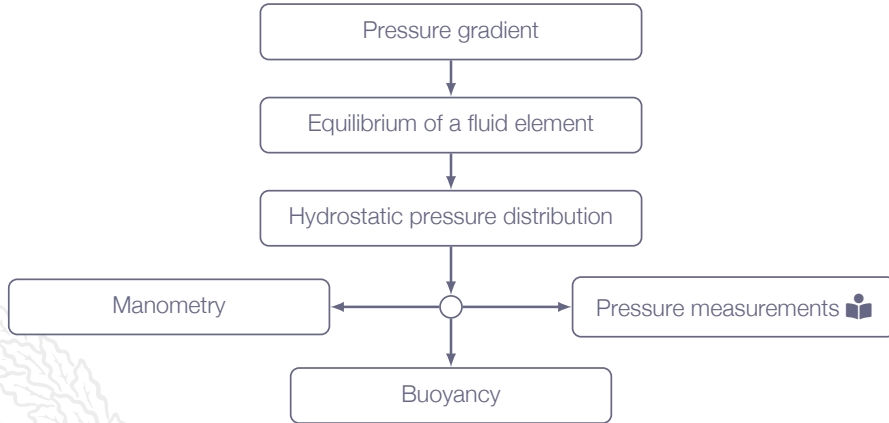
Learning Outcomes

- 9 **Explain** how to do a force balance for fluid element (forces and pressure gradients)
- 10 **Understand and explain** buoyancy and cavitation
- 11 **Solve** problems involving hydrostatic pressure and buoyancy

we will have a look at the pressure distribution in a fluid at rest, i.e. no flow yet...



Roadmap - Pressure Distribution in a Fluid



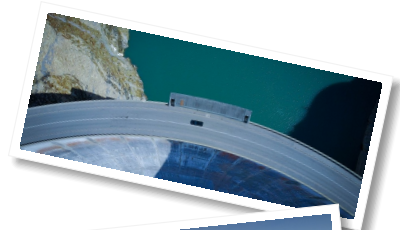
Motivation

Many problems does not include fluid motion

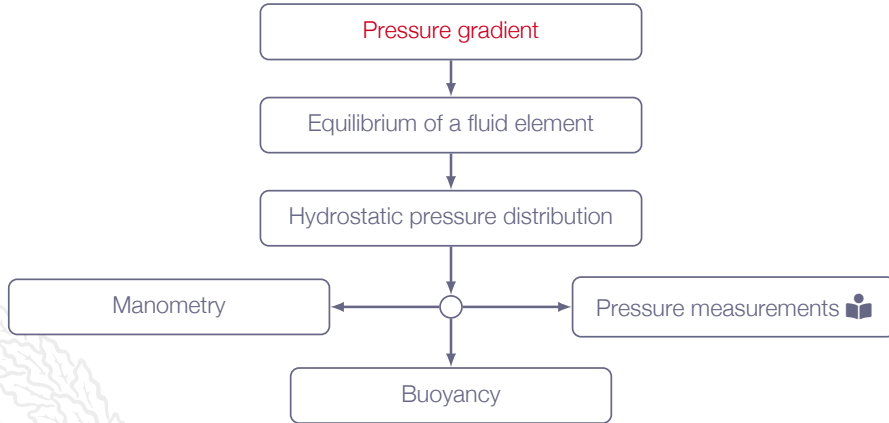
pressure distribution in a static fluid such as
the pressure in the atmosphere or in oceans

pressure on solid surfaces due to presence
of static fluid

floating and submerged bodies



Roadmap - Pressure Distribution in a Fluid



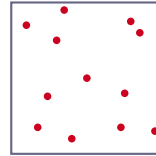
Pressure

Pressure is a thermodynamic property

Pressure is not a force and has no direction

Forces arise when the molecules of the fluid interacts with the surface of an immersed body

A force in the surface-normal direction is generated due to the collision of fluid molecules and the surface

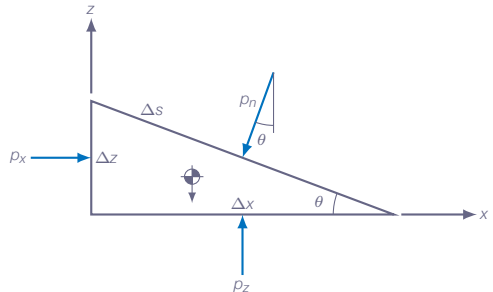


Pressure Variation in a Fluid at Rest

Fluid at rest - no shear (by definition)

Pressures p_x , p_z , and p_n may be different

Small element $\Rightarrow$ constant pressure on each face

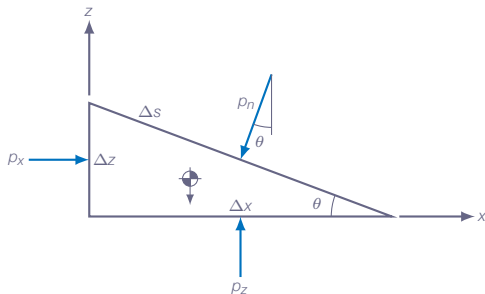


Pressure Variation in a Fluid at Rest

$$\sum F_x = 0 = p_x b \Delta z - p_n b \Delta s \sin \theta$$

$$\sum F_z = 0 = p_z b \Delta x - p_n b \Delta s \cos \theta - \frac{1}{2} \rho g b \Delta x \Delta z$$

$$\begin{cases} \Delta z = \Delta s \sin \theta \\ \Delta x = \Delta s \cos \theta \end{cases}$$

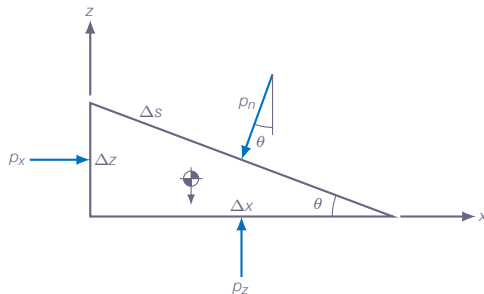


Pressure Variation in a Fluid at Rest

$$\sum F_x = 0 = p_x b \Delta z - p_n b \Delta z$$

$$\sum F_z = 0 = p_z b \Delta x - p_n b \Delta x - \frac{1}{2} \rho g b \Delta x \Delta z$$

$$\begin{cases} p_x = p_n \\ p_z = p_n + \frac{1}{2} \rho g \Delta z \end{cases}$$



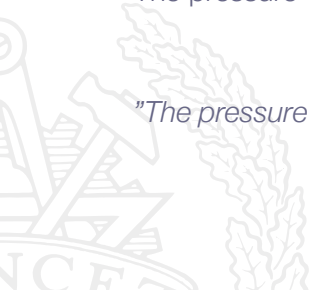
Pressure Variation in a Fluid at Rest

Since θ is arbitrary, the result is general

There is no pressure change in the horizontal direction

The pressure change in the vertical direction is proportional to the depth

"The pressure in a static fluid is a point property, independent of orientation"



Pressure Forces on a Fluid Element

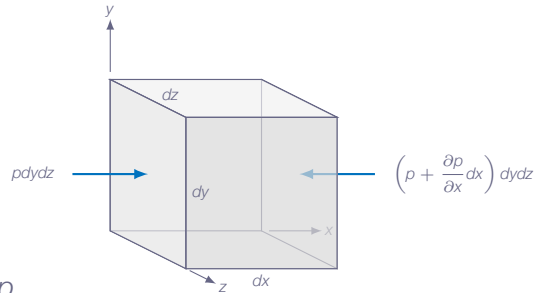
$$p = p(x, y, z, t)$$

$$dF_x = p dydz - \left(p + \frac{\partial p}{\partial x} dx \right) dydz = -\frac{\partial p}{\partial x} dx dydz$$

$$d\mathbf{F}_p = - \left[\frac{\partial p}{\partial x} \mathbf{e}_x + \frac{\partial p}{\partial y} \mathbf{e}_y + \frac{\partial p}{\partial z} \mathbf{e}_z \right] dx dy dz$$

$$\mathbf{f}_p = -\nabla p$$

$\mathbf{f}$ is the net force per unit volume

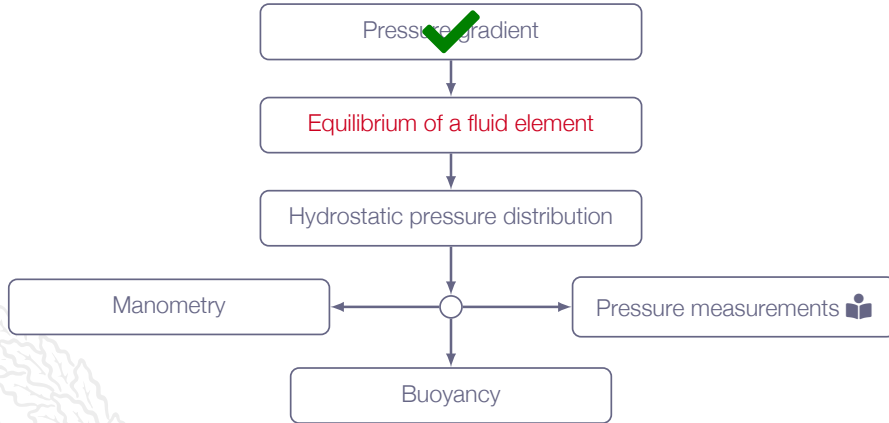


Pressure Forces on a Fluid Element

*"it is not the pressure but the **pressure gradient** causing a net force which must be balanced by gravity or acceleration"*



Roadmap - Pressure Distribution in a Fluid



Equilibrium of a Fluid Element

Force balance for a small element

pressure gradients gives surface forces

body forces (electromagnetic or gravitational potentials)

surface forces due to viscous stresses

Newton's second law:

$$\sum \mathbf{f} = \mathbf{f}_p + \mathbf{f}_g + \mathbf{f}_v = -\nabla p + \rho \mathbf{g} + \mathbf{f}_v = \rho \mathbf{a}$$

Equilibrium of a Fluid Element

Hydrostatic problems:

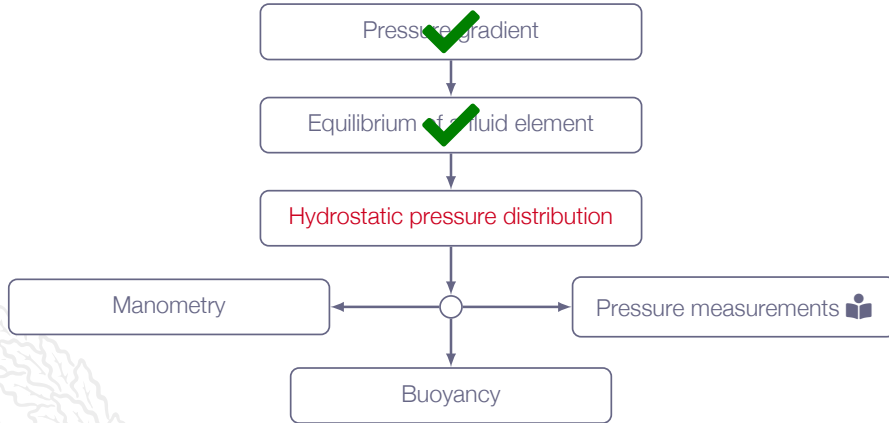
1. no viscous forces
2. no acceleration

Newton's second law reduces to:

$$\nabla p = \rho \mathbf{g}$$

(the general form of Newton's second law will be studied later)

Roadmap - Pressure Distribution in a Fluid



Hydrostatic Pressure in Liquids

$$\nabla p = \rho \mathbf{g}$$

∇p is perpendicular everywhere to surfaces of constant p

The normal of constant-pressure surfaces will be aligned with $\mathbf{g}$



Hydrostatic Pressure in Liquids

$$\mathbf{g} = -g\mathbf{e}_z$$

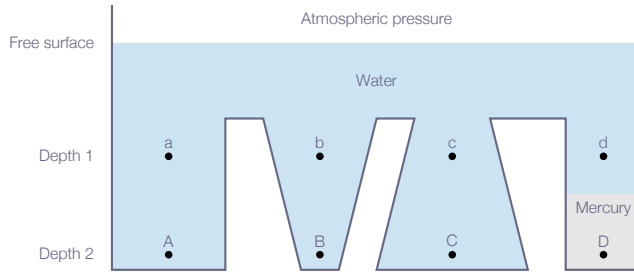
$$\frac{dp}{dz} = -\rho g$$

$$p_2 - p_1 = -\int_1^2 \rho g dz$$



Hydrostatic Pressure in Liquids

for liquids, we assume constant density $\Rightarrow p_2 - p_1 = - \int_1^2 \rho g dz = -\rho g(z_2 - z_1)$



$$p_a = p_b = p_c = p_d$$
$$p_A = p_B = p_C \neq p_D$$

Hydrostatic Pressure in Liquids

Is the incompressible assumption for liquids a good assumption?

the density is 4.6 percent higher at the deepest part of the ocean - so yes!

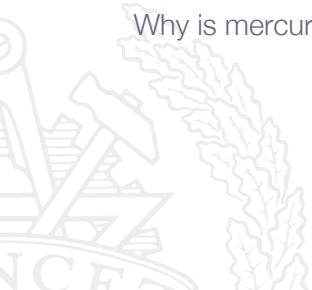
$$p_2 - p_1 = - \int_1^2 \rho g dz = -\rho g(z_2 - z_1)$$



Hydrostatic Pressure in Liquids

$$p_2 - p_1 = - \int_1^2 \rho g dz = -\rho g(z_2 - z_1)$$

Why is mercury used for pressure measurements?



Hydrostatic Pressure in Gases

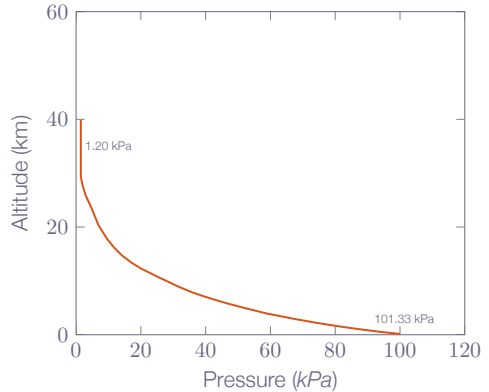
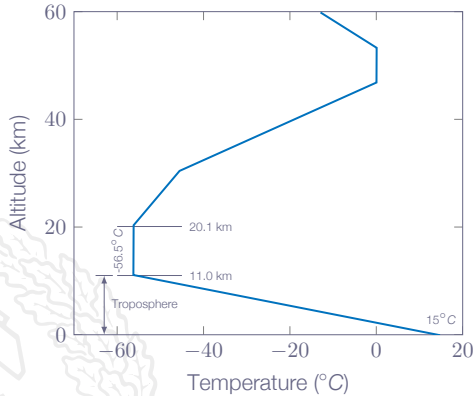
$$\frac{dp}{dz} = -\rho g = -\frac{p}{RT}g$$

both pressure and temperature varies with altitude

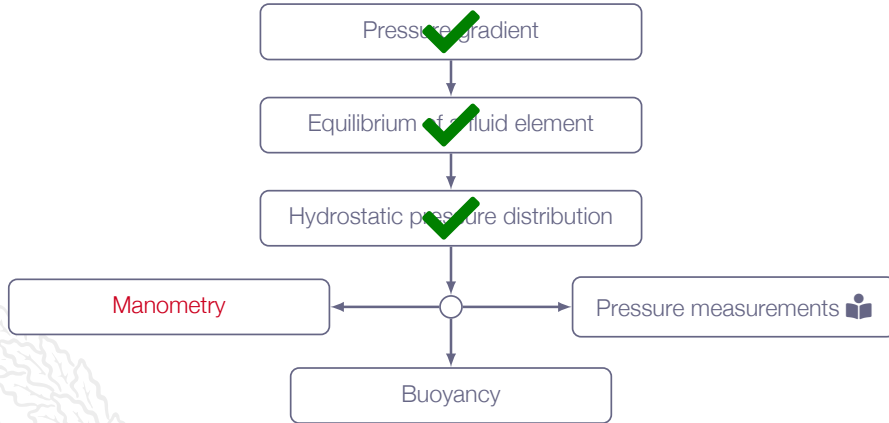
$$\int_1^2 \frac{dp}{p} = \ln \frac{p_2}{p_1} = -\frac{g}{R} \int_1^2 \frac{dz}{T}$$

Temperature variation $T(z)$ needed

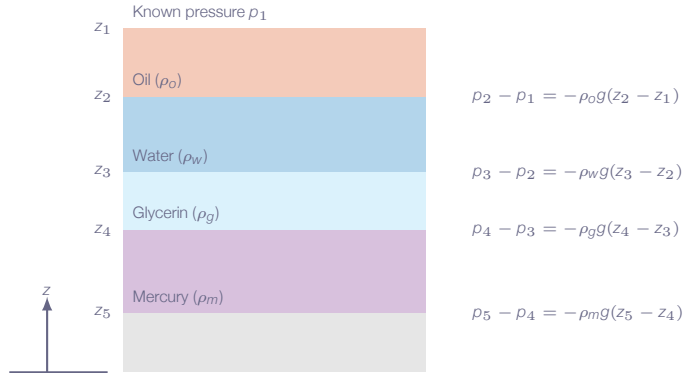
Hydrostatic Pressure in Gases



Roadmap - Pressure Distribution in a Fluid

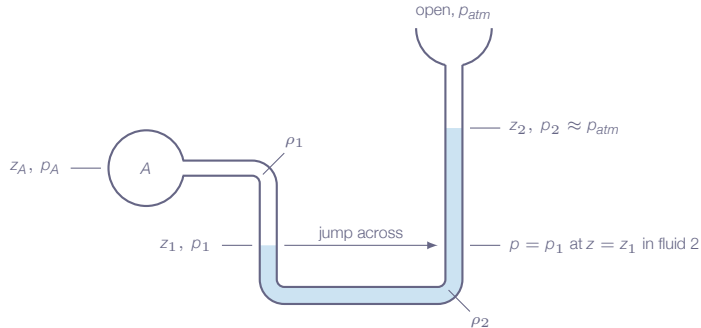


Manometry



$$p_5 - p_1 = -\rho_m g(z_5 - z_4) - \rho_g g(z_4 - z_3) - \rho_w g(z_3 - z_2) - \rho_o g(z_2 - z_1)$$

Manometry

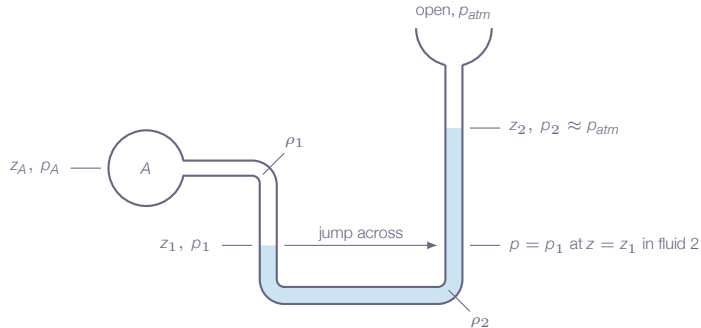


Pascal's law:

"Any two points at the same elevation in a continuous mass of the same static fluid will be at the same pressure"

Manometry

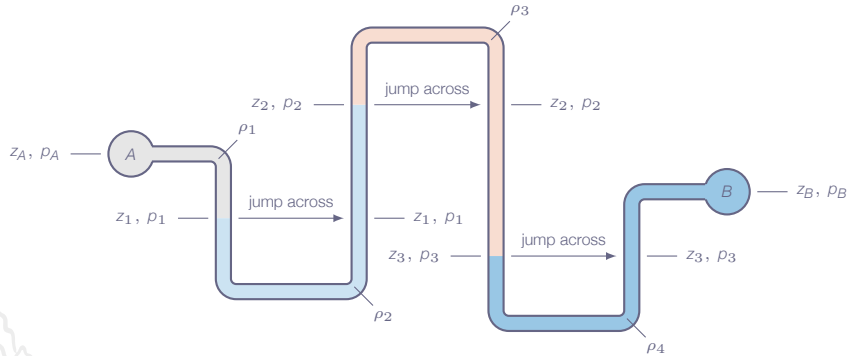
$$p_A + \rho_1 g(z_A - z_1) - \rho_2 g(z_2 - z_1) = p_2 \approx p_{atm}$$



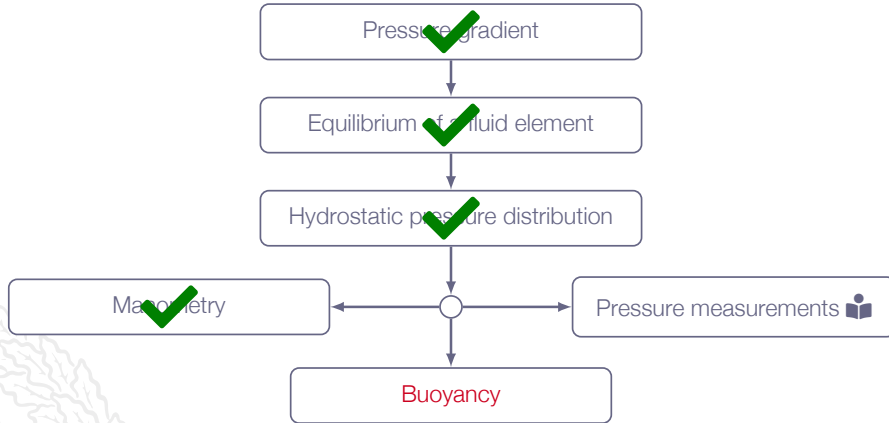
Pascal's law:

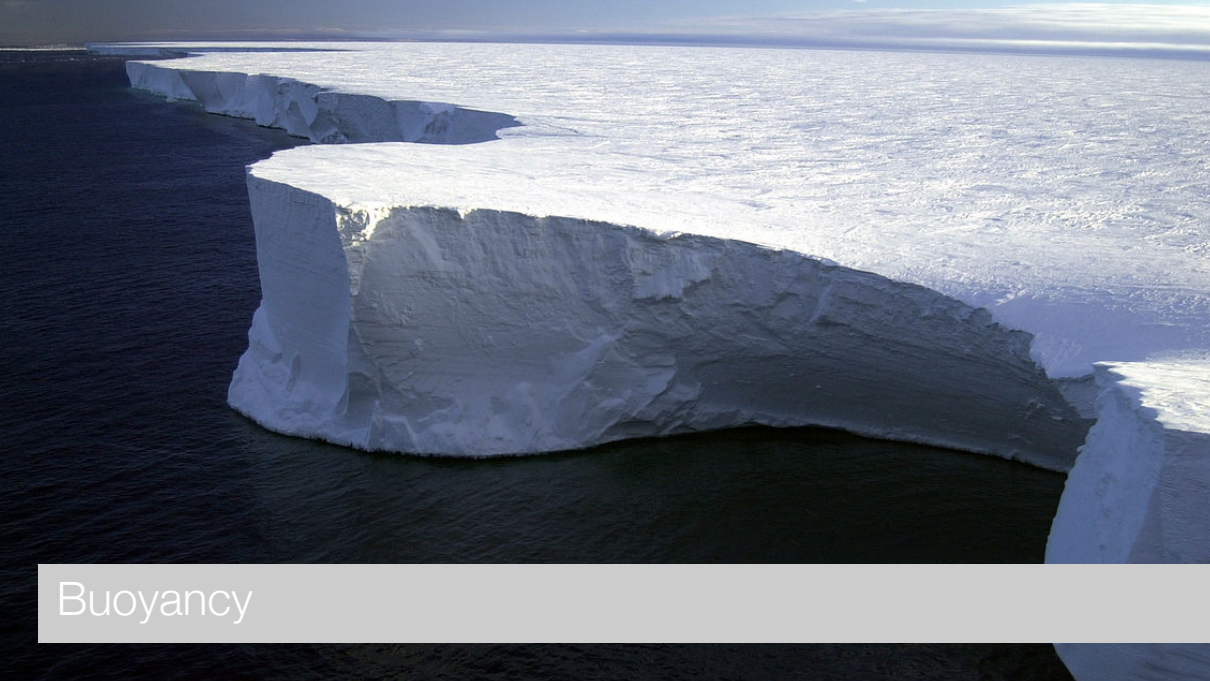
"Any two points at the same elevation in a continuous mass of the same static fluid will be at the same pressure"

Manometry



Roadmap - Pressure Distribution in a Fluid





Buoyancy

Buoyancy

Archimedes:

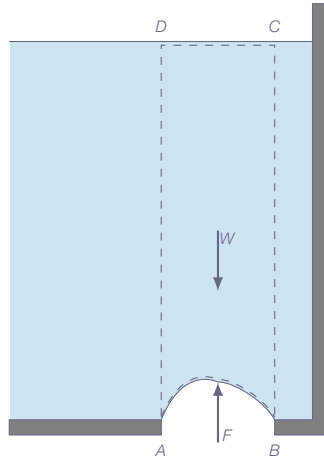


A body immersed in a fluid experiences a vertical buoyant force equal to the **weight of the fluid it displaces**

A floating body **displaces its own weight** in the fluid in which it floats

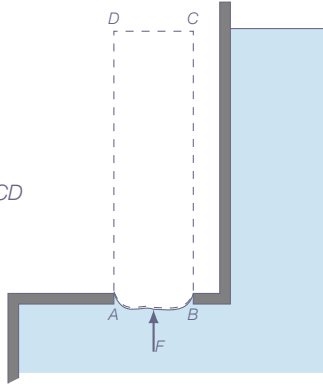
Buoyancy

$$F = \rho g \mathcal{V}_{ABCD}$$

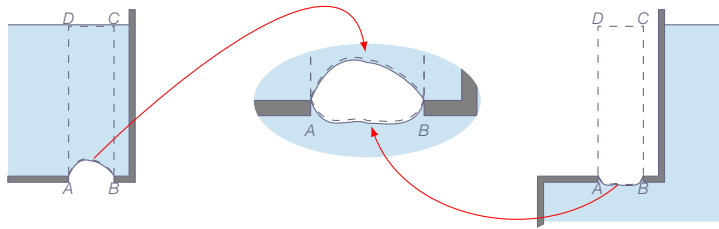


Buoyancy

$$F = \rho g V_{ABCD}$$



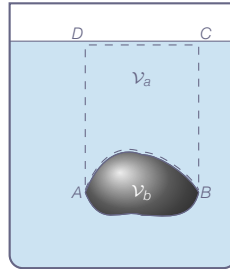
Buoyancy



Buoyancy

$$F_{up} = \rho g(\nu_a + \nu_b)$$

$$F_{down} = \rho g \nu_a$$



Buoyancy

In general

$$\mathbf{F}_B = \sum \rho_i g (\text{displacement volume})_i$$

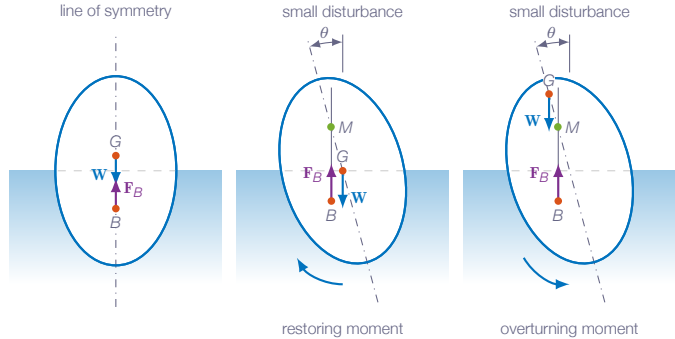
Floating bodies

$$\mathbf{F}_B = \text{body weight}$$



Buoyancy - Stability

Center of gravity G
Center of buoyancy B
Symmetry line
Metacenter M

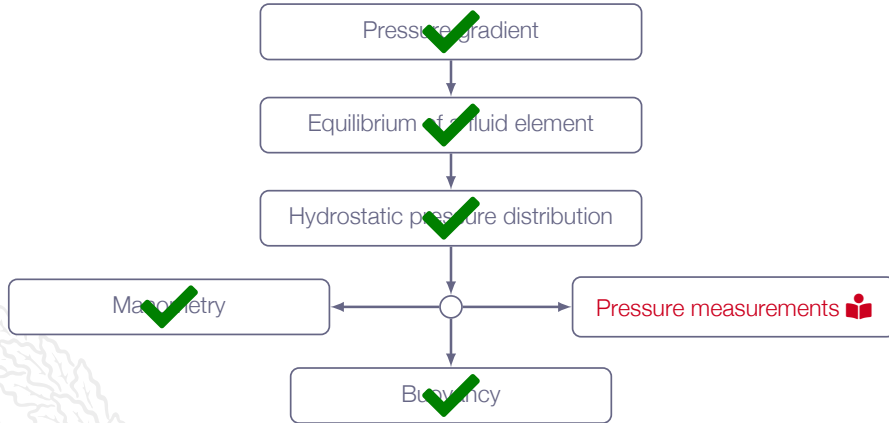


Note! the center of buoyancy (B) is, in this case, the centroid of the displaced volume of liquid

Buoyancy - Stability



Roadmap - Pressure Distribution in a Fluid



Pressure measurement

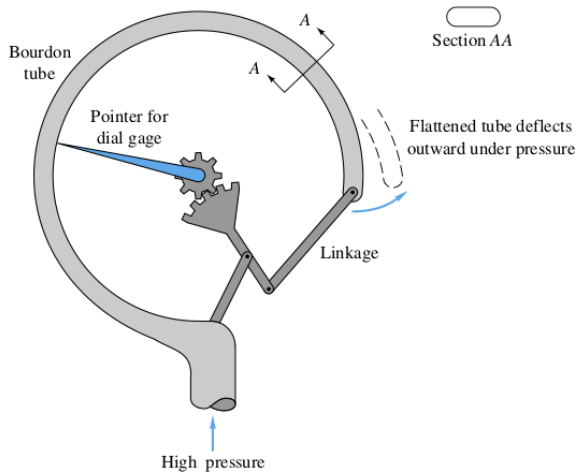


Pressure is a derived property

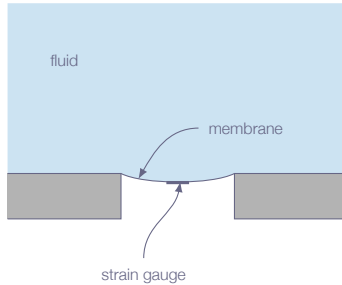
The force per unit area related to fluid molecular bombardment of a surface



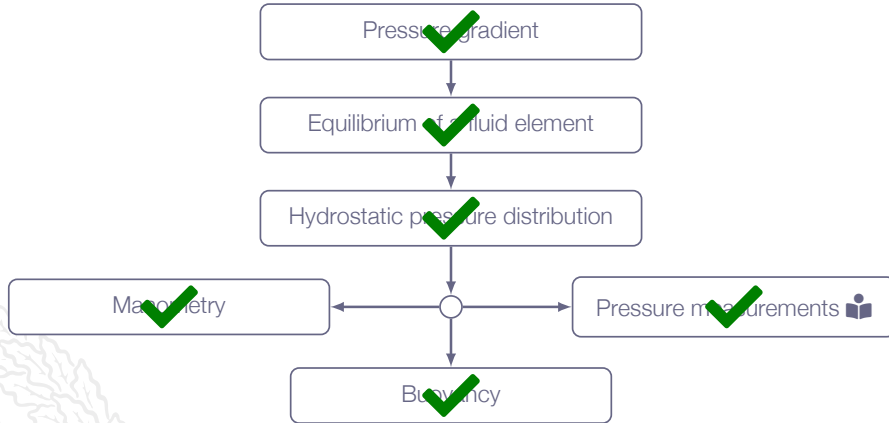
Pressure measurement



Pressure measurement



Roadmap - Pressure Distribution in a Fluid

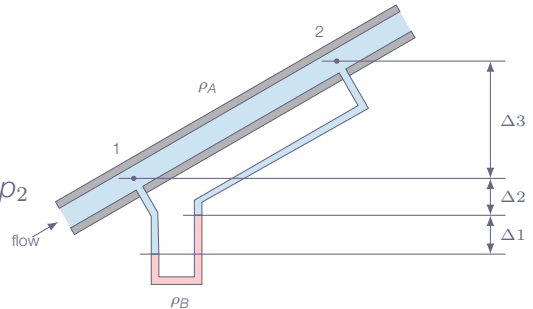


Manometer Example

$$p_1 + \sum_{\text{down}} \rho_i g \Delta_i - \sum_{\text{up}} \rho_i g \Delta_i = p_2$$

$$p_1 + (\Delta_2 + \Delta_1) \rho_A g - \Delta_1 \rho_B g - (\Delta_2 + \Delta_3) \rho_A g = p_2$$

$$p_1 + (\Delta_2 + \Delta_1) \rho_A g - \Delta_1 \rho_B g - (\Delta_2 + z_2 - z_1) \rho_A g = p_2$$



$$\left(\frac{p_1}{\rho_A g} + z_1 \right) - \left(\frac{p_2}{\rho_A g} + z_2 \right) = \Delta_1 \left(\frac{\rho_B}{\rho_A} - 1 \right)$$

Iceberg Efficiency

