

TME085 - Compressible Flow

2015-03-19, 08.30-13.30, M-building

Approved aids:

- *TME085 Compressible Flow - Formulas, tables and graphs* (provided with exam)
- *Beta - Mathematics Handbook for Science and Engineering*
- Optional calculator/Valfri miniräknare (graph drawing calculators with cleared memory allowed)

Grading:

number of points on exam	24-35	36-47	48-60
grade	3	4	5

Note that at least 6 points needs to be in the theoretical part (Part I) and at least 10 points in the problem part (Part II)

Part I - Theory Questions (20 p.)

T1. (1 p.)

What conditions must be satisfied for a steady-state compressible flow to be isentropic?

T2. (1 p.)

For a steady-state adiabatic compressible flow of calorically perfect gas, which of the variables p_0 (total pressure) and T_0 (total temperature) is/are constant along streamlines? Why?

T3. (1 p.)

Derive the relation

$$\frac{T_0}{T} = 1 + \frac{1}{2}(\gamma - 1)M^2$$

for calorically perfect gas from the energy equation form

$$h_0 = h + \frac{1}{2}V^2.$$

T4. (1 p.)

The oblique shock generated by a two-dimensional wedge in a supersonic steady-state flow can be either of the “*weak*” type or the “*strong*” type. What is the main difference between these two shock types and which type is usually seen in reality?

T5. (1 p.)

High temperature effects in compressible flows are found when analyzing for example very strong shocks or nozzle flows with extremely high total pressure and total enthalpy. What is the root cause of these effects and what do we mean by equilibrium gas? What kind of thermodynamic relations are needed to compute the flow of equilibrium gas?

T6. (1 p.)

In shock tubes, unsteady contact discontinuities are sometimes found. Describe in words what they are and under which circumstances they may be formed. Which of the variables p , T , ρ , u , s is/are necessarily continuous across such a contact discontinuity?

T7. (1 p.)

A stationary normal shock with upstream Mach number M_1 ($M_1 > 1$) is compared to a moving normal shock, traveling with Mach number M_S into quiescent (non-moving) air. If $M_1 = M_S$, is there any physical difference between the two shock waves apart from the fact that they have different speeds relative to the observer?

T8. (1 p.)

What is the difference between a calorically perfect gas and a thermally perfect gas?

T9. (1 p.)

What is meant by an under-expanded or over-expanded nozzle flow?

T10. (2 p.)

Derive the special formula for the speed of sound a for a calorically perfect gas:

$$a = \sqrt{\gamma RT}$$

starting from the general formula:

$$a^2 = \left(\frac{\partial p}{\partial \rho} \right)_s$$

T11. (1 p.)

In steady-state 2D supersonic flow there are two types of shock reflection at solid walls. Name these two reflection types and describe the difference between them.

T12. (1 p.)

What is the difference between acoustic waves and other types of waves such as shock waves and expansion waves?

T13. (2 p.)

Derive the area-velocity relation in quasi-1D flow starting from the mass conservation relation

$$d(\rho u A) = 0.,$$

Euler's equation

$$dp = -\rho u du,$$

and the definition of the speed of sound

$$a^2 = \left(\frac{\partial p}{\partial \rho} \right)_s$$

What are the implications of the area-velocity relation for quasi-one-dimensional flow?

T14. (1 p.)

When applying a time-marching flow solution scheme the so-called CFL number is an important parameter. Define the CFL number and describe its significance. What is a typical maximum CFL number for an explicit time stepping scheme.

T15. (1 p.)

What is meant by the terms “*density-based*” and “*fully coupled*” when discussing CFD codes for compressible flow?

T16. (2 p.)

Describe in words the significance of Crocco's equation.

Prove that a steady-state irrotational flow with constant total enthalpy must also be isentropic.

T17. (1 p.)

How can we use our knowledge of characteristics (and their speed of propagation) to guide us when determining suitable boundary conditions for compressible flows?

Part II - Problems (40 p.)

Problem 1 - Convergent Nozzle (10 p.)

A circular convergent nozzle has an inlet radius of 0.2 [m] and an outlet radius of 0.1 [m]. The total temperature in the nozzle plenum chamber (T_o) is 300 [K]. If the static pressure at the nozzle exit is 1.0 [bar], calculate the plenum total pressure (p_o) such that the nozzle exit Mach number is 0.68. For the specified nozzle geometry and operating conditions, calculate the inlet conditions (Mach number, pressure, temperature, density, and axial velocity). The gas flowing through the nozzle is air (calorically perfect). The radial and tangential velocity components are zero and viscous effects may be neglected.

Solution:

Since we have inviscid subsonic flow we can treat the process as isentropic
Using the relation

$$\frac{p_o}{p} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\gamma/(\gamma-1)}$$

and the exit pressure and Mach number we get $p_o = 1.3628$ [bar]

In order to get the nozzle inlet conditions, we must use the area-Mach-number relation

At the nozzle outlet we have from table A.1 that $A_2/A^* = 1.11$ which means that we can calculate A^* ($A^* = 0.028$ [m^2])

The inlet area (or radius) is given and from that we get $A_1/A^* = 4.44$

Using A_1/A^* , from table A.1 (or by solving the area-velocity relation) we get the Mach number at the nozzle inlet $M_1 = 0.13$

Again, using the relations

$$\frac{p_o}{p} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\gamma/(\gamma-1)}$$

and

$$\frac{T_o}{T} = \left(1 + \frac{\gamma - 1}{2} M^2\right)$$

gives us the pressure and temperature at the inlet
we get the density from the ideal gas law

$$p = \rho RT$$

and the speed of sound

$$a = \sqrt{\gamma RT}$$

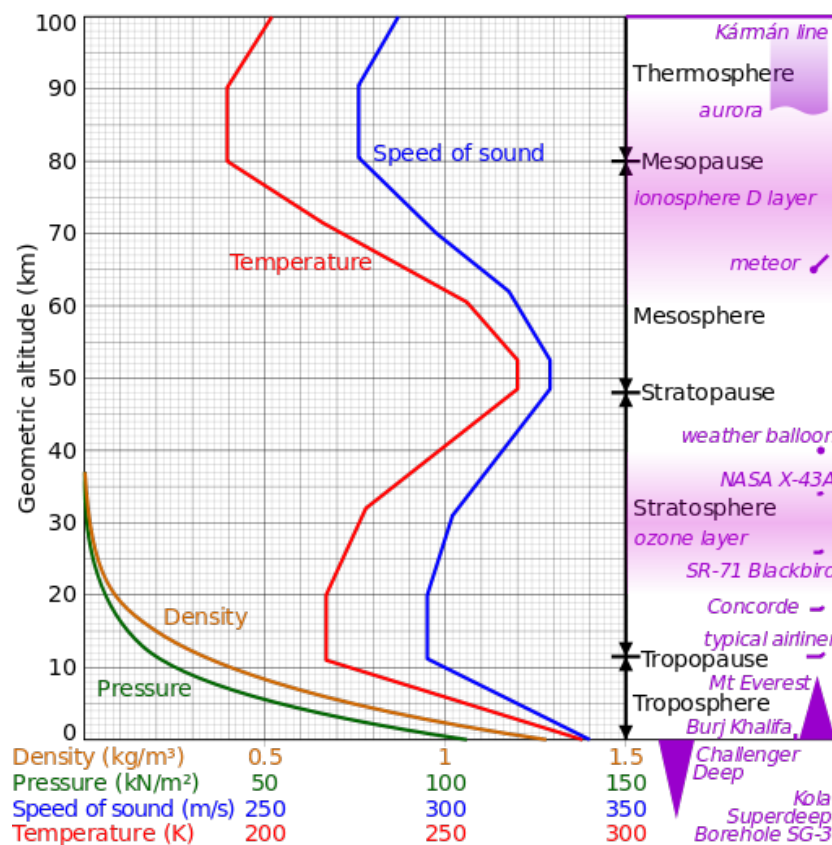
gives us the inlet velocity $u = Ma$

p_1	1.35	[bar]
T_1	299.0	[K]
ρ_1	1.57	[kg/m ³]
M_1	0.13	
u_1	45.7	[m/s]

Problem 2 - Engine Intake (10 p.)

A supersonic aircraft flies at a Mach number of 2.7 at an altitude of 20 km. The air that enters the engine inlet is slowed down isentropically to a Mach number of 1.3 in the engine intake. At that location, a normal shock occurs. The resulting flow is decelerated adiabatically, but not isentropically, further to a Mach number of 0.4. The final static pressure is 104 kPa.

- Calculate:
 - the stagnation temperature for the freestream flow
 - the pressure change over the shock Δp
 - the final stagnation pressure
 - the total entropy change for the entire process
- Sketch the deceleration process in a T - s -diagram



Atmospheric variations of thermodynamic quantities (Wikipedia)

Solution:

We need the static pressure and temperature at the specified altitude. From the provided figure we get $T \approx 217$ [K] and $p \approx 5.5$ [kPa]

using

$$\frac{T_o}{T} = \left(1 + \frac{\gamma - 1}{2} M^2\right)$$

with $M=2.7$ we get $T_o = 533.4$ [K]

Next we want to calculate the pressure change over a normal shock located a position in the engine intake where the flow has been slowed down isentropically to a Mach number of 1.3 here we use the normal shock relation

$$\frac{p_2}{p_1} = 1 + \frac{2\gamma}{\gamma + 1}(M_1^2 - 1)$$

$$\Delta p = p_2 - p_1 = \frac{p_2}{p_1} p_1 - p_1 = p_1 \frac{2\gamma}{\gamma + 1}(M_1^2 - 1)$$

we know that before this point we have isentropic deceleration and thus we can calculate p_1 from p_o and the Mach number

$$\frac{p_o}{p} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\gamma/(\gamma-1)}$$

Using this we get $\Delta p = 37$ [kPa]

The final total pressure we get from the final static pressure ($p = 104$ [kPa]) and the final Mach number ($M = 0.4$)

$$\frac{p_o}{p} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\gamma/(\gamma-1)}$$

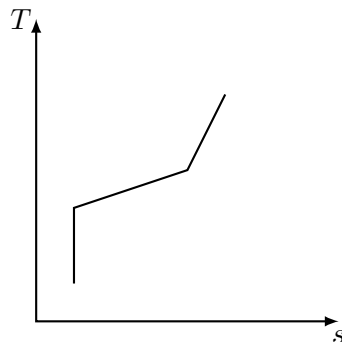
gives $p_o = 116.1$ [kPa]

The total entropy change can be, since the process is adiabatic, be calculated from

$$\Delta s = -R \ln \frac{p_{o2}}{p_{o1}}$$

which gives $\Delta s = 26.3$ [J/(kg K)]

In a T - s -diagram, the process would look something like:



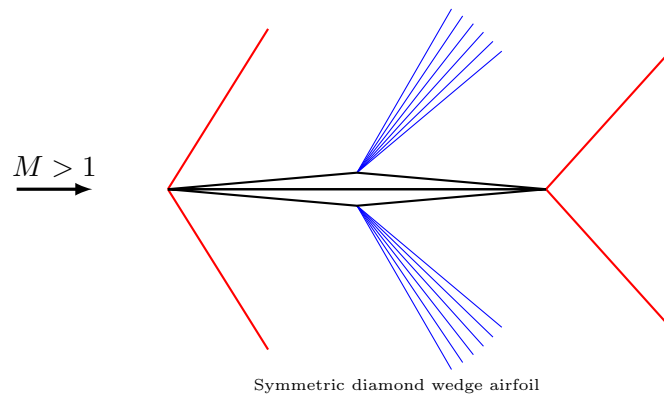
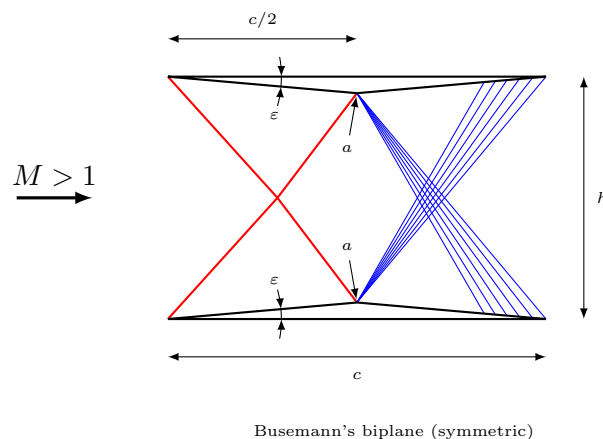
Problem 3 - Busemann's Biplane (10 p.)



<http://newsoffice.mit.edu/2012/supersonic-biplane-0319>

Busemann's Biplane is a conceptual airframe design invented by Adolf Busemann in the 1950's. The wing construction is, as a system, under design conditions free of external shock waves thus reducing the sonic boom effects. The concept is based on a symmetric diamond wedge airfoil that is split in half. The two halves are flipped upside down (the flat sides are parallel to the fluid flow) and separated a distance h . The spacing between the plates is sufficiently large so that the flow does not choke and thus supersonic flow is maintained through the passage.

- (a) With a half-tip angle (ε) of 5 degrees, a wing chord length (c) of 1.0 [m], and a freestream Mach number of 1.5, calculate the wing separation h such that the reflected shock reaches the airfoil at point a (design condition). The flowing medium is air (calorically perfect).
- (b) Sketch the pressure distribution on the upper surface on the lower wing section of Busemann's biplane concept (exact numbers not needed)



Solution:

To solve this problem, we use the θ - β - M -relation combined with Mach number relations for oblique shocks to calculate the angles for the two shocks. After that it's all about geometry and trigonometry.

First shock:

The Mach number before the first oblique shock is given ($M_1=1.5$). Together with the deflection angle ($\theta=\varepsilon=5.0$ degrees) and the θ - β - M -relation we get the angle $\beta_1 \approx 48$ degrees

Now, we need to calculate the Mach number after the shock (for the flow aligned with the wing surface)

first we calculate the normal Mach number of the incident flow

$$M_{n_1} = M_1 \sin(\beta_1)$$

than we calculate the normal Mach number after the first shock

$$M_{n_2} = \sqrt{\frac{M_{n_1}^2 + [2/(\gamma - 1)]}{[2\gamma/(\gamma - 1)] M_{n_1}^2 - 1}}$$

The Mach number of the flow behind the shock is now calculated according to

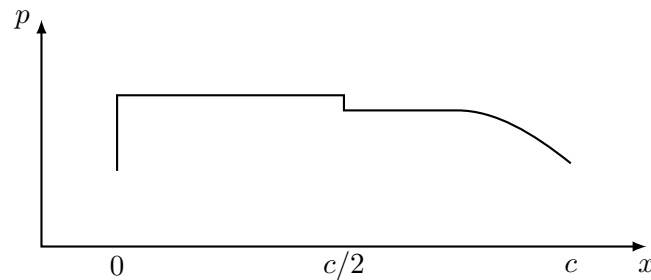
$$M_2 = M_{n_2} / \sin(\beta_1 - \varepsilon)$$

Second shock:

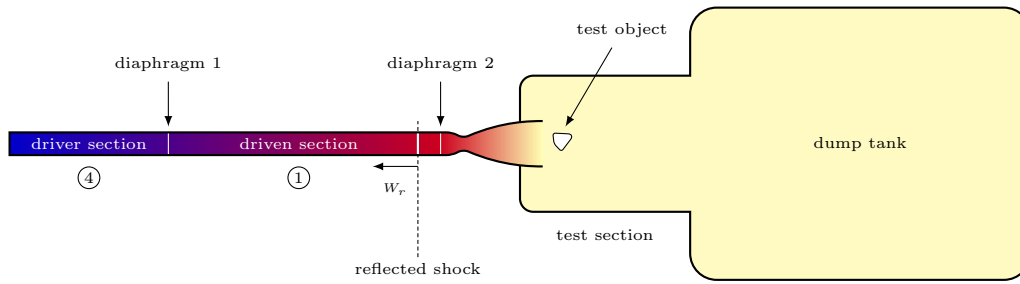
Since the flow after the second shock will be aligned with the flow before the first shock (due to symmetry constraints), the flow will again be deflected with an angle of $\varepsilon=5.0$ degrees. With the Mach number (M_2) and the θ - β - M -relation we get $\beta_2 \approx 58$ degrees

Using the geometric constraints that the two shocks must meet at $y=h/2$ and that the second shock intersects with the wing at the point $a=(c/2, t = c/2 \tan(\varepsilon))$, the distance h is calculated to be 0.64 [m]

The pressure distribution on the upper surface of the lower wing segment would look something like

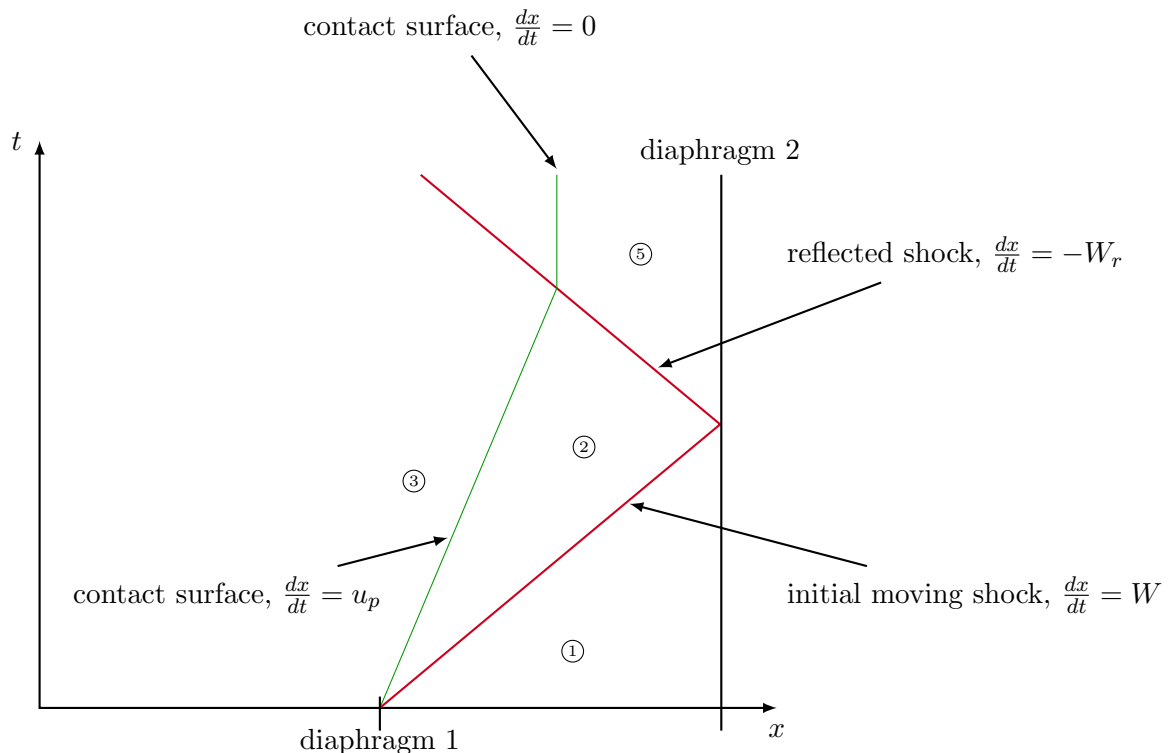


Problem 4 - Shock Tunnel (10 p.)



A shock tunnel is a combination of a shock tube and a convergent-divergent nozzle. When the traveling shock wave, generated as the diaphragm separating state 1 (driven gas) and state 4 (driver gas) breaks, reflects at the right end wall (diaphragm 2), high stagnation temperatures and pressures are achieved. These conditions are used as the plenum conditions for a convergent-divergent nozzle and thereby establish a high-Mach-number high-temperature flow (for a very short time).

- If the pressure ratio over the incident shock (before reflection) p_2/p_1 is 11.0 and the pressure (p_1) and temperature (T_1) in the driven section is 1.0 [bar] and 300 [K], calculate the nozzle plenum conditions ($T_o = T_5$, $p_o = p_5$)
- What back pressure (test chamber pressure) gives a Mach number of 4.0 at the nozzle exit (assume shock free nozzle flow). Both driver gas, driven gas and the gas in the test chamber is air (assumed to be calorically perfect).
- Comment on the results and the assumptions made.



Solution:

The shock tube relations are used to get the conditions at the right end of the shock tube after the shock reflection (T_5, p_5).

First the mach number for the incident shock is calculated using the relation

$$M_s = \sqrt{\frac{\gamma + 1}{2\gamma} \left(\frac{p_2}{p_1} - 1 \right)} + 1$$

Using this Mach number ($M_s=3.1$), we can get the Mach number of the reflected shock from the relation

$$\frac{M_R}{M_R^2 - 1} = \frac{M_s}{M_s^2 - 1} \sqrt{1 + \frac{2(\gamma - 1)}{(\gamma + 1)^2} (M_s^2 - 1) \left(\gamma + 1 \frac{1}{M_s^2} \right)}$$

Which results in a quadratic problem that has to be solved ($M_R=2.1$)

Now, using the normal shock relations for the reflected shock (the velocity in region 5 is zero and by changing frame of reference we can use these relations)

$$\frac{p_5}{p_2} = 1 + \frac{2\gamma}{\gamma + 1} (M_R^2 - 1)$$

$$\frac{T_5}{T_2} = \left[1 + \frac{2\gamma}{\gamma + 1} (M_R^2 - 1) \right] \left[\frac{2 + (\gamma - 1)M_R^2}{(\gamma + 1)M_R^2} \right]$$

We also need T_2/T_1 , which we get from

$$\frac{T_2}{T_1} = \frac{p_2}{p_1} \left[\frac{\frac{\gamma + 1}{\gamma - 1} + \frac{p_2}{p_1}}{1 + \frac{\gamma + 1}{\gamma - 1} \left(\frac{p_2}{p_1} \right)} \right]$$

The conditions in region 5 can now be calculated according to

$$p_5 = \frac{p_5}{p_2} \frac{p_2}{p_1} p_1$$

$$T_5 = \frac{T_5}{T_2} \frac{T_2}{T_1} T_1$$

which gives $p_5=56.3$ [bar] and $T_5=1502.6$ [K]

In the second part of the problem, the back pressure (test chamber pressure) in order to get a nozzle exit Mach number of 4.0 (for shock free nozzle conditions) is asked for

Shock free operation gives isentropic flow and we can use

$$\frac{p_o}{p} = \left(1 + \frac{\gamma - 1}{2} M^2 \right)^{\gamma/(\gamma-1)}$$

to get the pressure $p=0.37$ [bar]

Comments on the results and the assumptions made:

Here one can think of many things to comment and there is no correct answer examples of relevant comments can be

- We get a temperature of 1500 [K] and we use a calorically perfect gas assumption - this gas model is not really applicable in this range of temperatures
- We assume the nozzle flow to be shock free, which is a operating condition that is very difficult to achieve. In a real situations we would most likely have to deal with shocks
- The nozzle flow is probably very transient in a real life situation, which implies further complications