

Compressible Flow - TME085

Lecture 12

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Chapter 7

Unsteady Wave Motion



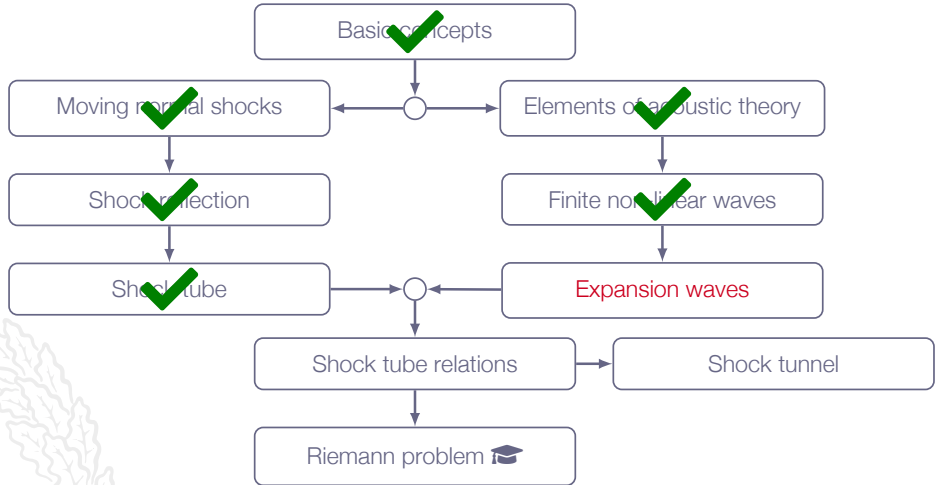


Learning Outcomes

- 3 **Describe** typical engineering flow situations in which compressibility effects are more or less predominant (e.g. Mach number regimes for steady-state flows)
- 4 **Present** at least two different formulations of the governing equations for compressible flows and **explain** what basic conservation principles they are based on
- 8 **Derive** (marked) and **apply** (all) of the presented mathematical formulae for classical gas dynamics
 - a 1D isentropic flow*
 - b normal shocks*
 - j unsteady waves and discontinuities in 1D
 - k basic acoustics
- 9 **Solve** engineering problems involving the above-mentioned phenomena (8a-8k)
- 11 **Explain** how the equations for aero-acoustics and classical acoustics are derived as limiting cases of the compressible flow equations

moving normal shocks - frame of reference seems to be the key here?!

Roadmap - Unsteady Wave Motion

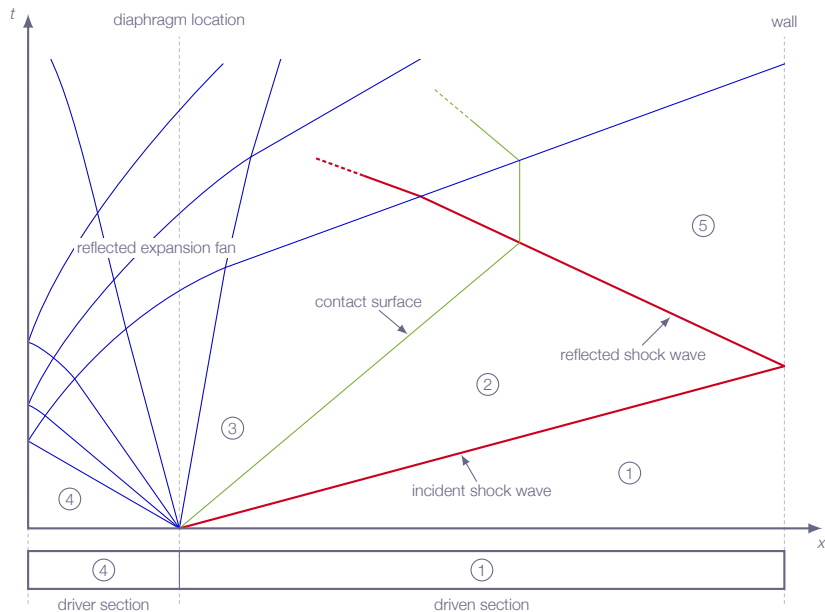


Chapter 7.7

Incident and Reflected Expansion Waves



Expansion Waves



Expansion Waves

Properties of a left-running expansion wave

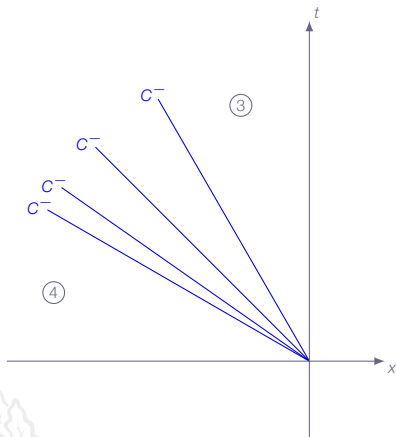
1. All flow properties are constant along C^- characteristics
2. The wave **head** is propagating **into region 4** (high pressure)
3. The wave **tail** defines the **limit of region 3** (lower pressure)
4. Regions 3 and 4 are assumed to be **constant states**

For calorically perfect gas:

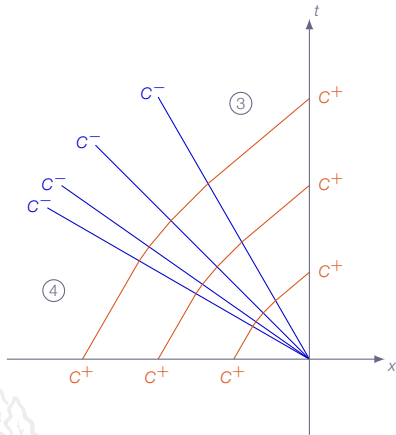
$$J^+ = u + \frac{2a}{\gamma - 1} \quad \text{is constant along } C^+ \text{ lines}$$

$$J^- = u - \frac{2a}{\gamma - 1} \quad \text{is constant along } C^- \text{ lines}$$

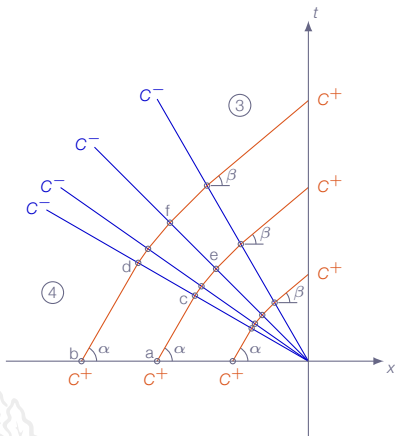
Expansion Waves



Expansion Waves



Expansion Waves



constant flow properties in region 4: $J_a^+ = J_b^+$

J^+ invariants constant along C^+ characteristics:

$$J_a^+ = J_c^+ = J_e^+$$

$$J_b^+ = J_d^+ = J_f^+$$

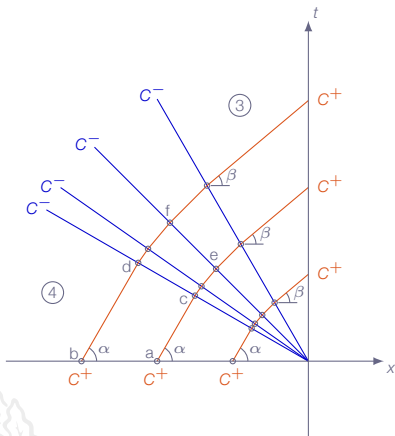
since $J_a^+ = J_b^+$ this also implies $J_e^+ = J_f^+$

J^- invariants constant along C^- characteristics:

$$J_c^- = J_d^-$$

$$J_e^- = J_f^-$$

Expansion Waves



constant flow properties in region 4: $J_a^+ = J_b^+$

J^+ invariants constant along C^+ characteristics:

$$J_a^+ = J_c^+ = J_e^+$$

$$J_b^+ = J_d^+ = J_f^+$$

since $J_a^+ = J_b^+$ this also implies $J_e^+ = J_f^+$

J^- invariants constant along C^- characteristics:

$$J_c^- = J_d^-$$

$$J_e^- = J_f^-$$

$$u_e = \frac{1}{2}(J_e^+ + J_e^-), u_f = \frac{1}{2}(J_f^+ + J_f^-), \Rightarrow u_e = u_f$$

$$a_e = \frac{\gamma - 1}{4}(J_e^+ - J_e^-), a_f = \frac{\gamma - 1}{4}(J_f^+ - J_f^-), \Rightarrow a_e = a_f$$

Expansion Waves

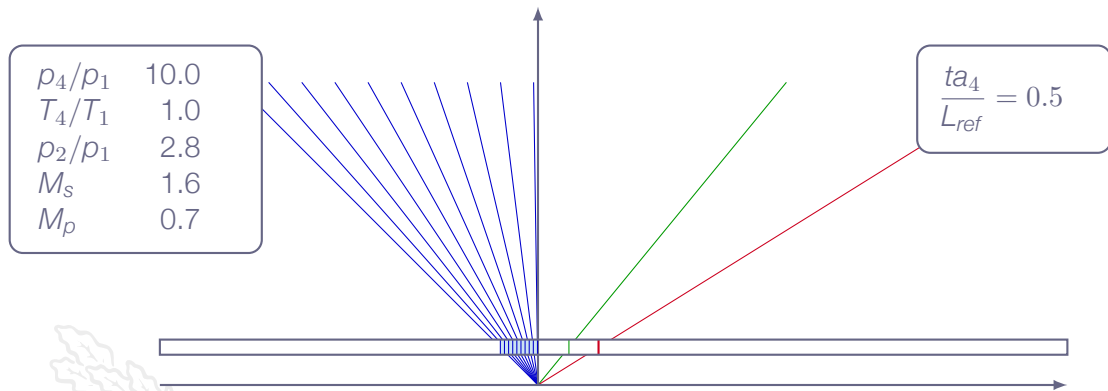
Along each C^- line u and a are **constants** which means that

$$\frac{dx}{dt} = u - a = \text{const}$$

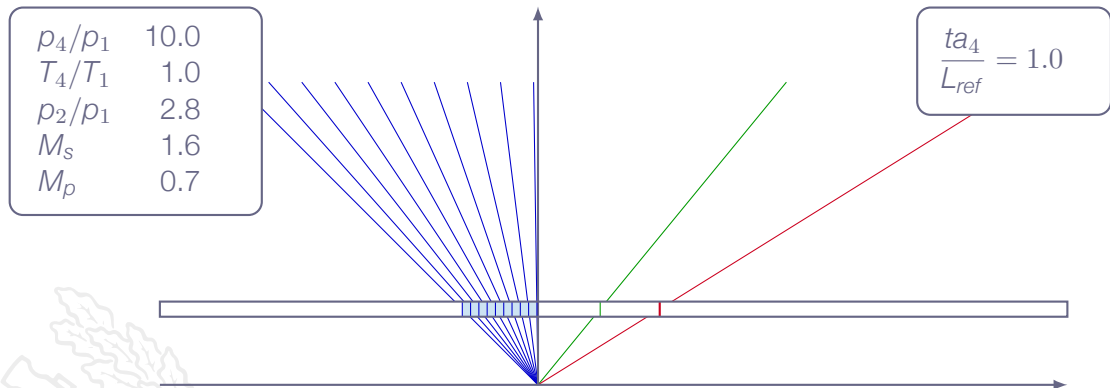
C^- characteristics are **straight lines** in xt -space



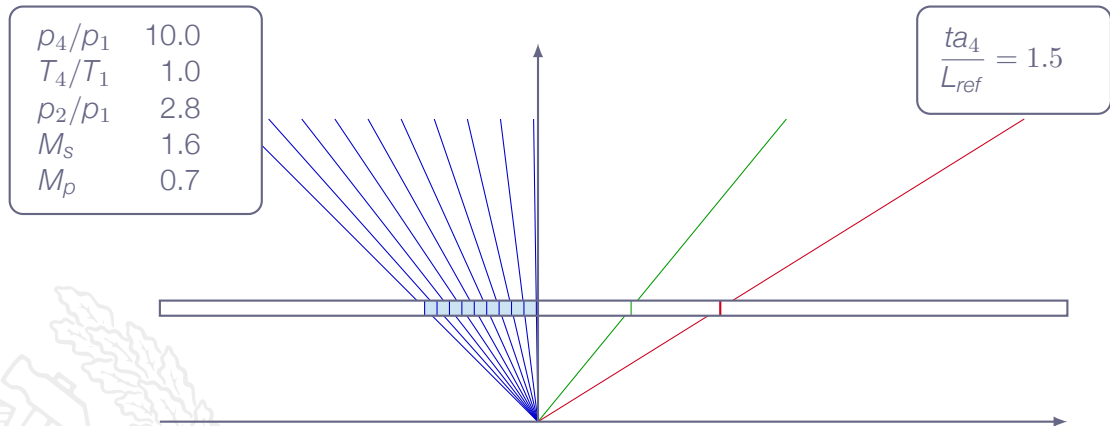
Expansion Waves - Shock Tube



Expansion Waves - Shock Tube



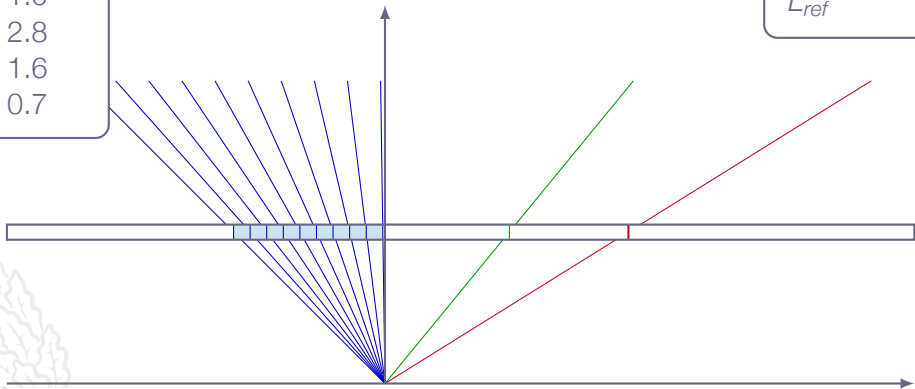
Expansion Waves - Shock Tube



Expansion Waves - Shock Tube

p_4/p_1	10.0
T_4/T_1	1.0
p_2/p_1	2.8
M_s	1.6
M_p	0.7

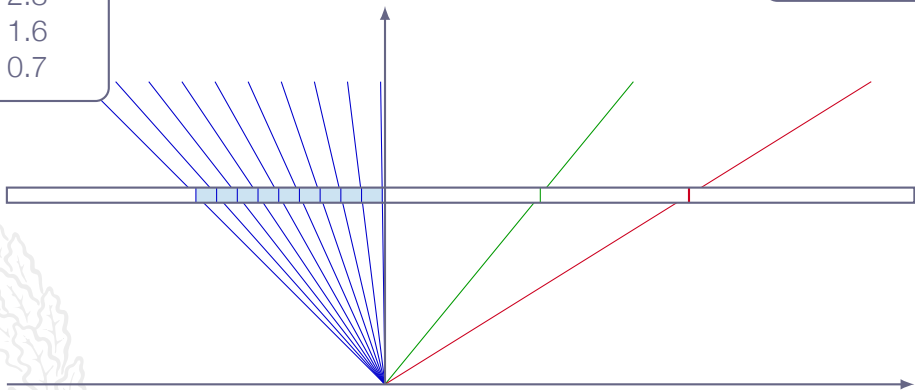
$$\frac{ta_4}{L_{ref}} = 2.0$$



Expansion Waves - Shock Tube

p_4/p_1	10.0
T_4/T_1	1.0
p_2/p_1	2.8
M_s	1.6
M_p	0.7

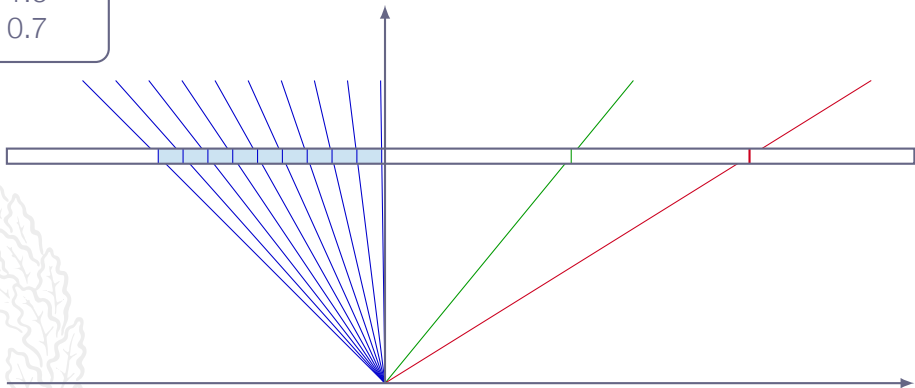
$$\frac{ta_4}{L_{ref}} = 2.5$$



Expansion Waves - Shock Tube

p_4/p_1	10.0
T_4/T_1	1.0
p_2/p_1	2.8
M_s	1.6
M_p	0.7

$$\frac{ta_4}{L_{ref}} = 3.0$$



Shock Tube Expansion Waves - Summary

The start and end conditions are the same for all C^+ lines

J^+ invariants have the same value for all C^+ characteristics

C^- characteristics are straight lines in xt -space

Simple expansion waves centered at $(x, t) = (0, 0)$

Expansion Waves

In a left-running expansion fan:

J^+ is constant throughout expansion fan, which implies:

$$u + \frac{2a}{\gamma - 1} = u_4 + \frac{2a_4}{\gamma - 1} = u_3 + \frac{2a_3}{\gamma - 1}$$

J^- is constant along C^- lines, but varies from one line to the next, which means that

$$u - \frac{2a}{\gamma - 1}$$

is constant along each C^- line

Expansion Waves

Since $u_4 = 0$ we obtain:

$$u + \frac{2a}{\gamma - 1} = u_4 + \frac{2a_4}{\gamma - 1} = \frac{2a_4}{\gamma - 1} \Rightarrow$$

$$\frac{a}{a_4} = 1 - \frac{1}{2}(\gamma - 1)\frac{u}{a_4}$$

with $a = \sqrt{\gamma RT}$ we get

$$\frac{T}{T_4} = \left[1 - \frac{1}{2}(\gamma - 1)\frac{u}{a_4} \right]^2$$

Expansion Wave Relations

Isentropic flow $\Rightarrow$ we can use the isentropic relations

complete description in terms of u/a_4

$$\frac{T}{T_4} = \left[1 - \frac{1}{2}(\gamma - 1) \frac{u}{a_4} \right]^2$$

$$\frac{\rho}{\rho_4} = \left[1 - \frac{1}{2}(\gamma - 1) \frac{u}{a_4} \right]^{\frac{2\gamma}{\gamma-1}}$$

$$\frac{\rho}{\rho_4} = \left[1 - \frac{1}{2}(\gamma - 1) \frac{u}{a_4} \right]^{\frac{2}{\gamma-1}}$$

Expansion Wave Relations

Since C^- characteristics are straight lines, we have:

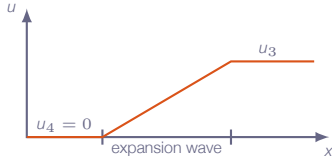
$$\frac{dx}{dt} = u - a \Rightarrow x = (u - a)t$$

$$\frac{a}{a_4} = 1 - \frac{1}{2}(\gamma - 1)\frac{u}{a_4} \Rightarrow a = a_4 - \frac{1}{2}(\gamma - 1)u \Rightarrow$$

$$x = \left[u - a_4 + \frac{1}{2}(\gamma - 1)u \right] t = \left[\frac{1}{2}(\gamma - 1)u - a_4 \right] t \Rightarrow$$

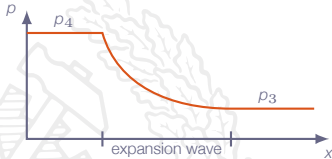
$$u = \frac{2}{\gamma + 1} \left[a_4 + \frac{x}{t} \right]$$

Expansion Wave Relations

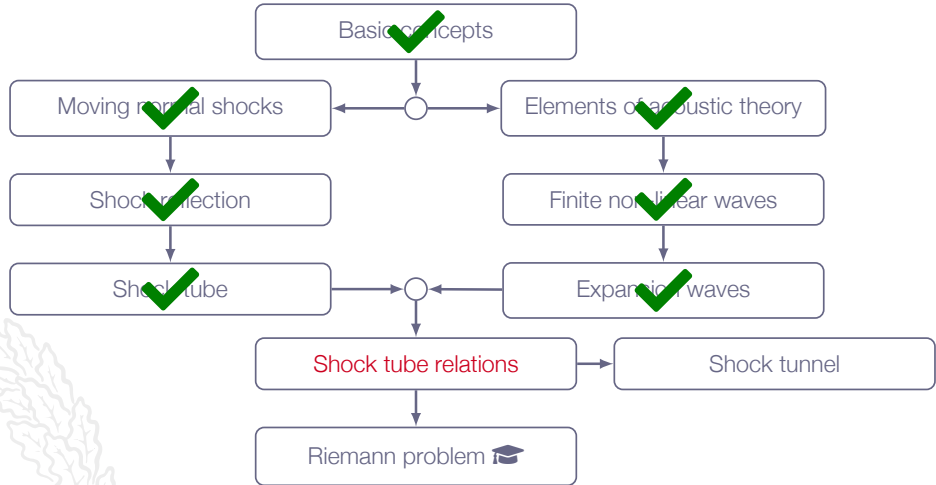


Expansion wave head is advancing to the left with speed a_4 into the stagnant gas

Expansion wave tail is advancing with speed $u_3 - a_3$, which may be positive or negative, depending on the initial states



Roadmap - Unsteady Wave Motion



Chapter 7.8

Shock Tube Relations



Shock Tube Relations

$$u_p = u_2 = \frac{a_1}{\gamma} \left(\frac{p_2}{p_1} - 1 \right) \left[\frac{\frac{2\gamma_1}{\gamma_1 + 1}}{\frac{p_2}{p_1} + \frac{\gamma_1 - 1}{\gamma_1 + 1}} \right]^{1/2}$$

$$\frac{p_3}{p_4} = \left[1 - \frac{\gamma_4 - 1}{2} \left(\frac{u_3}{a_4} \right) \right]^{2\gamma_4/(\gamma_4 - 1)}$$

solving for u_3 gives

$$u_3 = \frac{2a_4}{\gamma_4 - 1} \left[1 - \left(\frac{p_3}{p_4} \right)^{(\gamma_4 - 1)/(2\gamma_4)} \right]$$

Shock Tube Relations

But, $p_3 = p_2$ and $u_3 = u_2$ (no change in velocity and pressure over contact discontinuity)

$$\Rightarrow u_2 = \frac{2a_4}{\gamma_4 - 1} \left[1 - \left(\frac{p_2}{p_4} \right)^{(\gamma_4 - 1)/(2\gamma_4)} \right]$$

We have now two expressions for u_2 which gives us

$$\frac{a_1}{\gamma} \left(\frac{p_2}{p_1} - 1 \right) \left[\frac{\frac{2\gamma_1}{\gamma_1 + 1}}{\frac{p_2}{p_1} + \frac{\gamma_1 - 1}{\gamma_1 + 1}} \right]^{1/2} = \frac{2a_4}{\gamma_4 - 1} \left[1 - \left(\frac{p_2}{p_4} \right)^{(\gamma_4 - 1)/(2\gamma_4)} \right]$$

Shock Tube Relations

Rearranging gives:

$$\frac{p_4}{p_1} = \frac{p_2}{p_1} \left\{ 1 - \frac{(\gamma_4 - 1)(a_1/a_4)(p_2/p_1 - 1)}{\sqrt{2\gamma_1 [2\gamma_1 + (\gamma_1 + 1)(p_2/p_1 - 1)]}} \right\}^{-2\gamma_4/(\gamma_4 - 1)}$$

p_2/p_1 as implicit function of p_4/p_1

for a given p_4/p_1 , p_2/p_1 will increase with decreased a_1/a_4

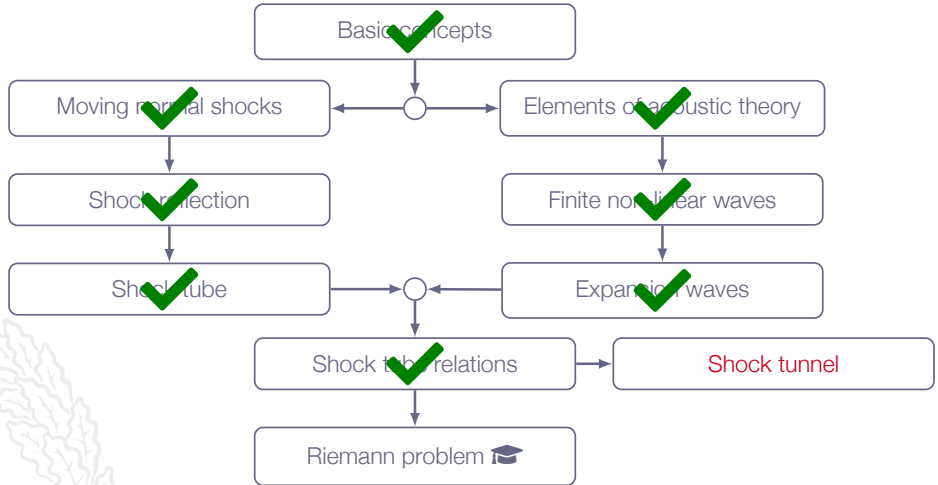
$$a = \sqrt{\gamma RT} = \sqrt{\gamma(R_u/M)T}$$

the speed of sound in a light gas is higher than in a heavy gas

driver gas: low molecular weight, high temperature

driven gas: high molecular weight, low temperature

Roadmap - Unsteady Wave Motion



Shock Tunnel

Addition of a convergent-divergent nozzle to a shock tube configuration

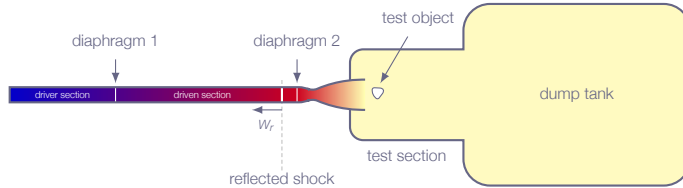
Capable of producing flow conditions which are close to those during the reentry of a space vehicles into the earth's atmosphere

high-enthalpy, hypersonic flows (short time)
real gas effects

Example - Aachen TH2:

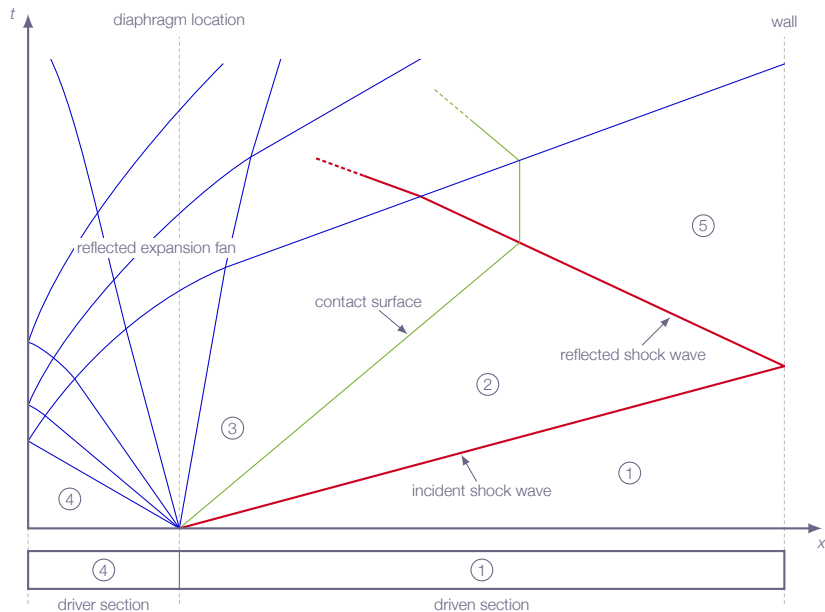
velocities up to 4 km/s
stagnation temperatures of several thousand degrees

Shock Tunnel



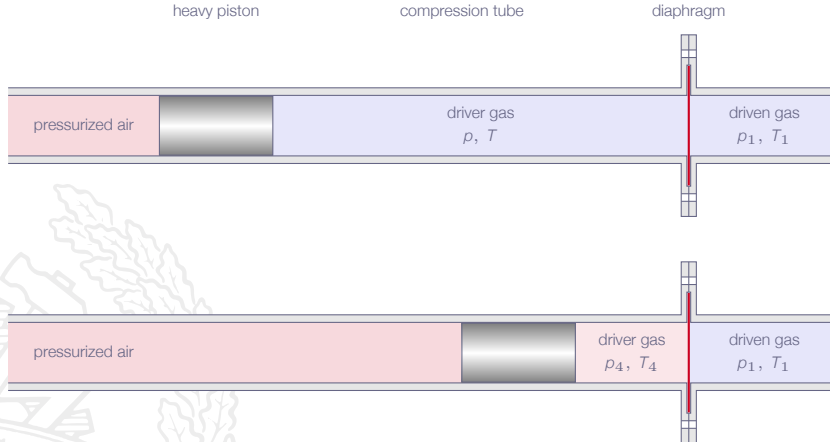
1. High pressure in region 4 (driver section)
diaphragm 1 burst
primary shock generated
2. Primary shock reaches end of shock tube
shock reflection
3. High pressure in region 5
diaphragm 2 burst
nozzle flow initiated
hypersonic flow in test section

Shock Tunnel

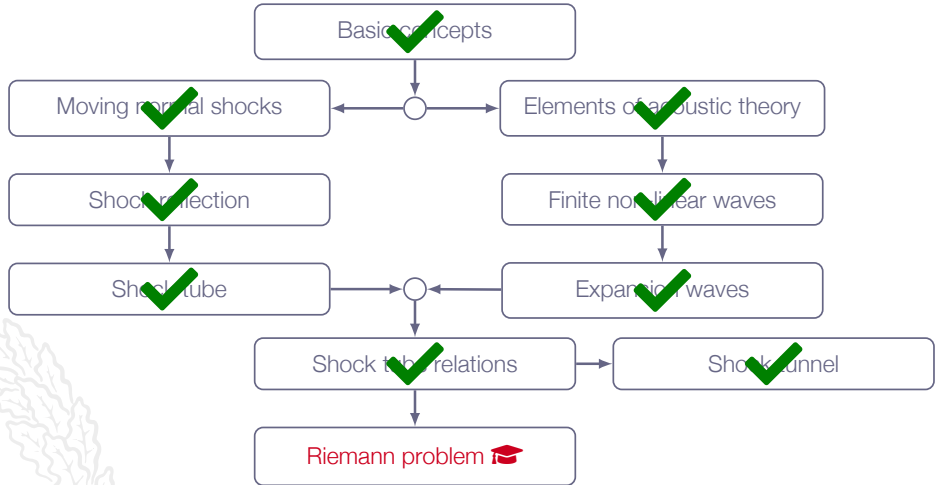


Shock Tunnel

By adding a compression tube to the shock tube a very high p_4 and T_4 may be achieved for any gas in a fairly simple manner



Roadmap - Unsteady Wave Motion





The shock tube problem is a special case of the general **Riemann Problem**

"... A Riemann problem, named after Bernhard Riemann, consists of an initial value problem composed by a conservation equation together with piece-wise constant data having a single discontinuity ..."

Wikipedia





May show that solutions to the shock tube problem have the general form:

$$p = p(x/t)$$

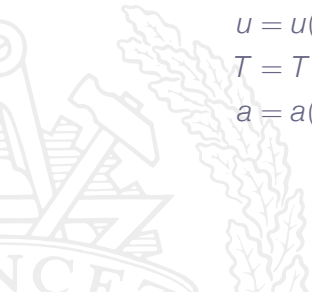
$$\rho = \rho(x/t)$$

$$u = u(x/t)$$

$$T = T(x/t)$$

$$a = a(x/t)$$

where $x = 0$ denotes the position of the initial jump between states 1 and 4





Numerical method:

Finite-Volume Method (FVM) solver

three-stage Runge-Kutta time stepping

third-order characteristic upwinding
scheme

local artificial damping

Left side conditions (state 4):

$$\rho = 2.4 \text{ kg/m}^3$$

$$u = 0.0 \text{ m/s}$$

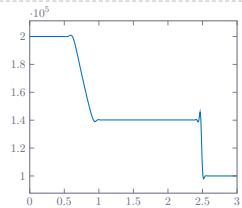
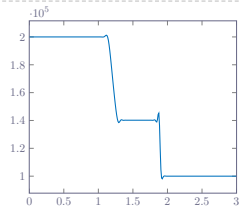
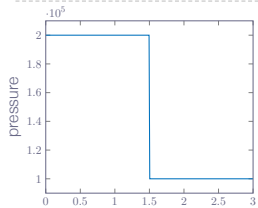
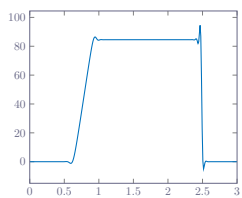
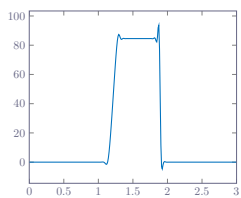
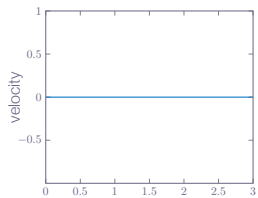
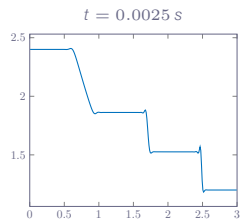
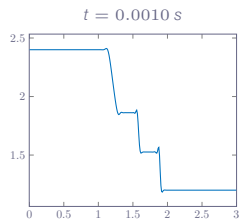
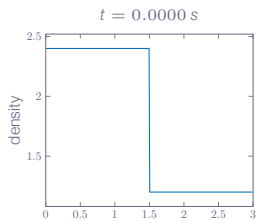
$$p = 2.0 \text{ bar}$$

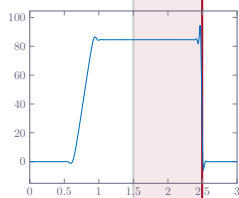
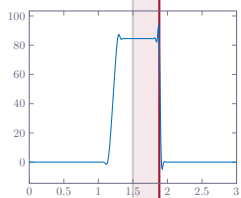
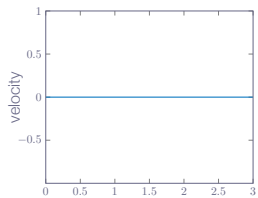
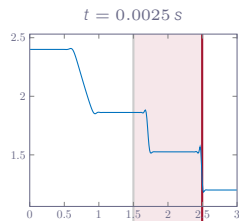
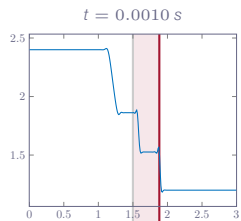
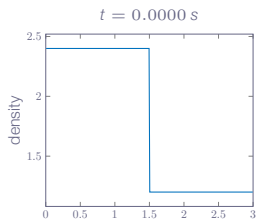
Right side conditions (state 1):

$$\rho = 1.2 \text{ kg/m}^3$$

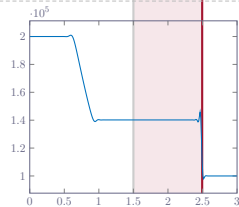
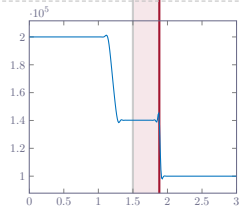
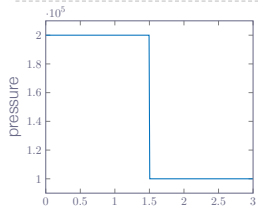
$$u = 0.0 \text{ m/s}$$

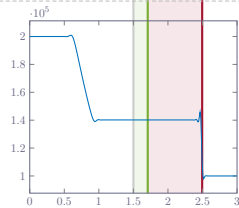
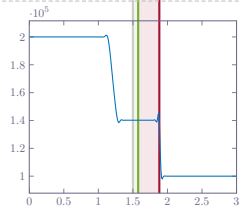
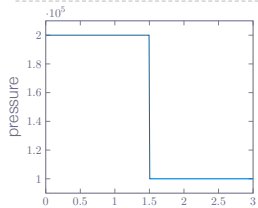
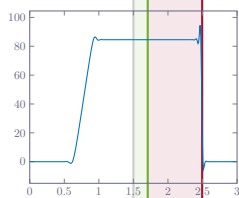
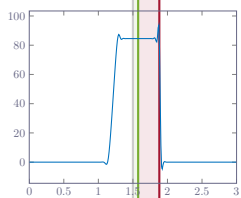
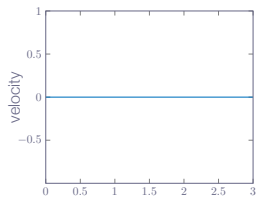
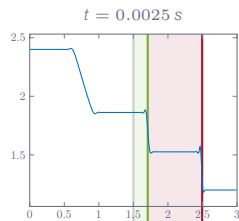
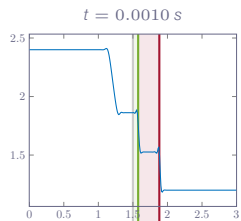
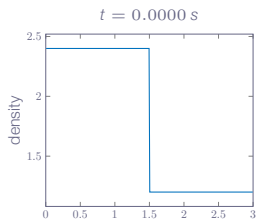
$$p = 1.0 \text{ bar}$$



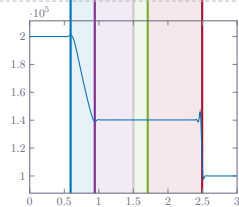
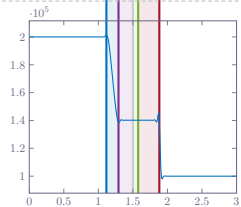
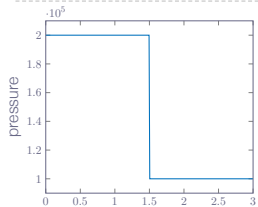
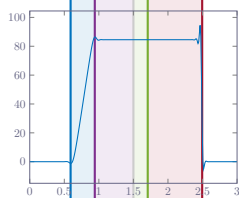
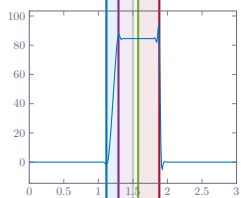
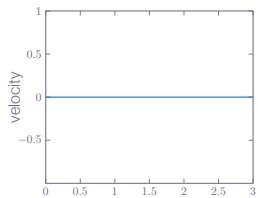
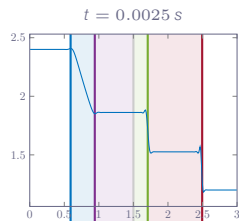
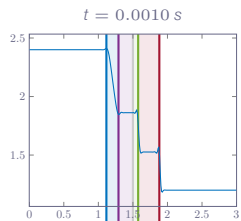
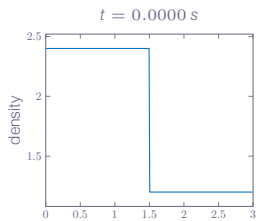


incident shock



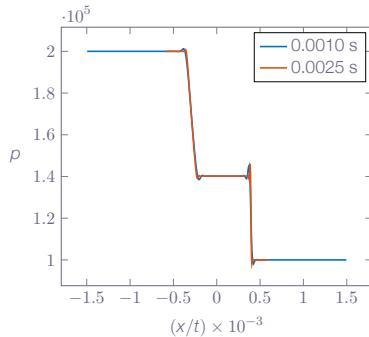
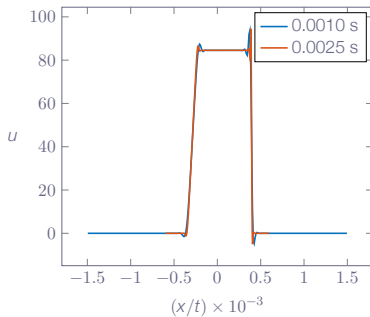
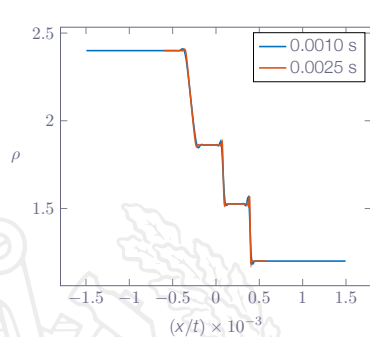


incident shock
contact discontinuity



incident shock
contact discontinuity
expansion wave

Riemann Problem - Shock Tube Simulation



The solution can be made self similar by plotting the flow field variables as function of x/t

Roadmap - Unsteady Wave Motion

