

Compressible Flow - TME085

Lecture 12

Niklas Andersson

Chalmers University of Technology
Department of Mechanical Engineering
Division of Fluid Mechanics
Gothenburg, Sweden

`niklas.andersson@chalmers.se`





Chapter 7 - Unsteady Wave Motion

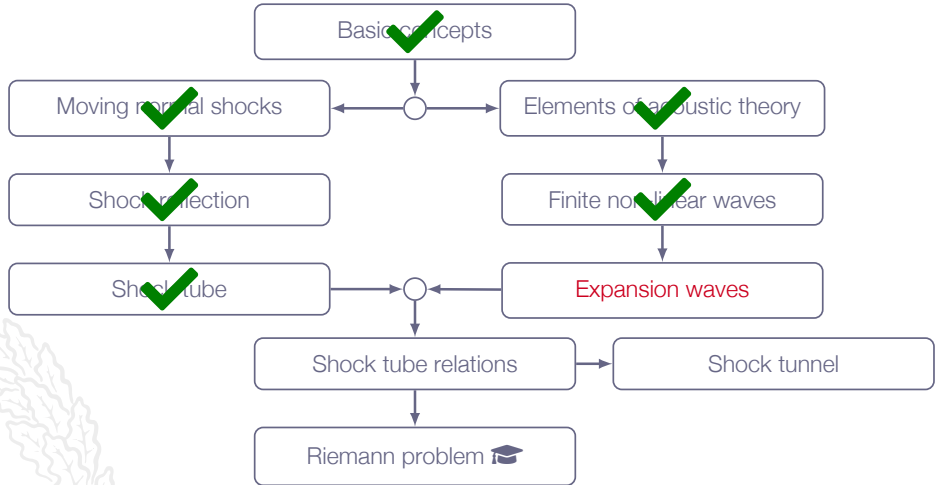


Learning Outcomes

- 3 **Describe** typical engineering flow situations in which compressibility effects are more or less predominant (e.g. Mach number regimes for steady-state flows)
- 4 **Present** at least two different formulations of the governing equations for compressible flows and **explain** what basic conservation principles they are based on
- 8 **Derive** (marked) and **apply** (all) of the presented mathematical formulae for classical gas dynamics
 - a 1D isentropic flow*
 - b normal shocks*
 - j unsteady waves and discontinuities in 1D
 - k basic acoustics
- 9 **Solve** engineering problems involving the above-mentioned phenomena (8a-8k)
- 11 **Explain** how the equations for aero-acoustics and classical acoustics are derived as limiting cases of the compressible flow equations

moving normal shocks - frame of reference seems to be the key here?!

Roadmap - Unsteady Wave Motion

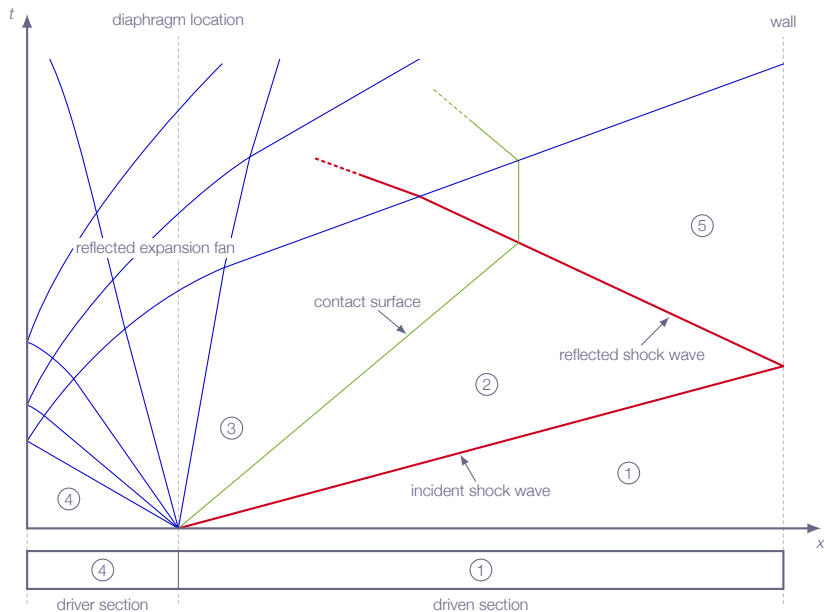


Chapter 7.7

Incident and Reflected Expansion Waves



Expansion Waves



Expansion Waves

Properties of a left-running expansion wave

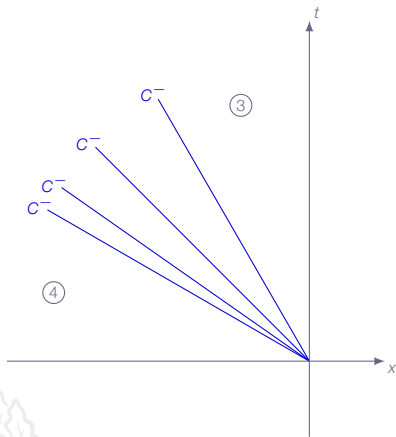
1. All flow properties are constant along C^- characteristics
2. The wave **head** is propagating **into region 4** (high pressure)
3. The wave **tail** defines the **limit of region 3** (lower pressure)
4. Regions 3 and 4 are assumed to be **constant states**

For calorically perfect gas:

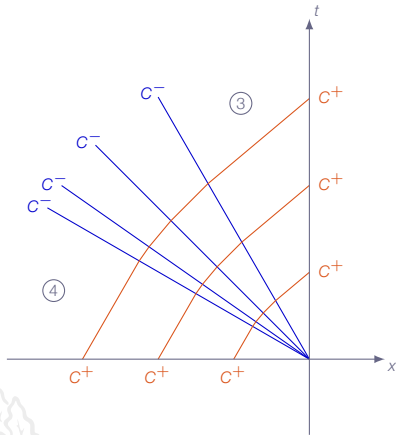
$$J^+ = u + \frac{2a}{\gamma - 1} \quad \text{is constant along } C^+ \text{ lines}$$

$$J^- = u - \frac{2a}{\gamma - 1} \quad \text{is constant along } C^- \text{ lines}$$

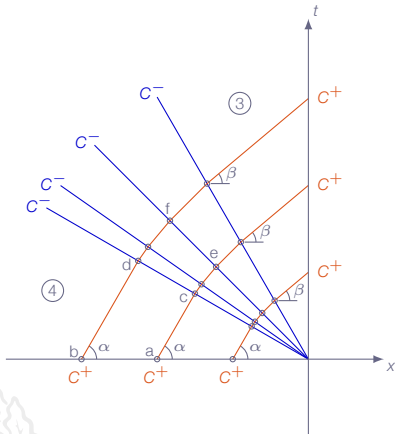
Expansion Waves



Expansion Waves



Expansion Waves



constant flow properties in region 4: $J_a^+ = J_b^+$

J^+ invariants constant along C^+ characteristics:

$$J_a^+ = J_c^+ = J_e^+$$

$$J_b^+ = J_d^+ = J_f^+$$

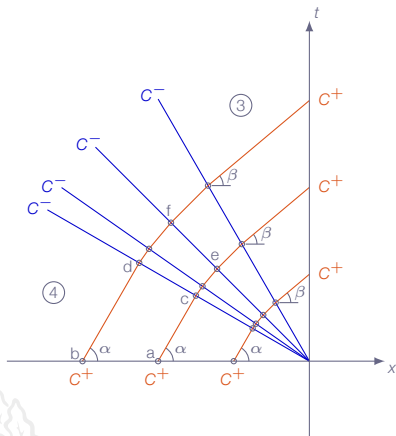
since $J_a^+ = J_b^+$ this also implies $J_e^+ = J_f^+$

J^- invariants constant along C^- characteristics:

$$J_c^- = J_d^-$$

$$J_e^- = J_f^-$$

Expansion Waves



constant flow properties in region 4: $J_a^+ = J_b^+$

J^+ invariants constant along C^+ characteristics:

$$J_a^+ = J_c^+ = J_e^+$$

$$J_b^+ = J_d^+ = J_f^+$$

since $J_a^+ = J_b^+$ this also implies $J_e^+ = J_f^+$

J^- invariants constant along C^- characteristics:

$$J_c^- = J_d^-$$

$$J_e^- = J_f^-$$

$$u_e = \frac{1}{2}(J_e^+ + J_e^-), u_f = \frac{1}{2}(J_f^+ + J_f^-), \Rightarrow u_e = u_f$$

$$a_e = \frac{\gamma - 1}{4}(J_e^+ - J_e^-), a_f = \frac{\gamma - 1}{4}(J_f^+ - J_f^-), \Rightarrow a_e = a_f$$

Expansion Waves

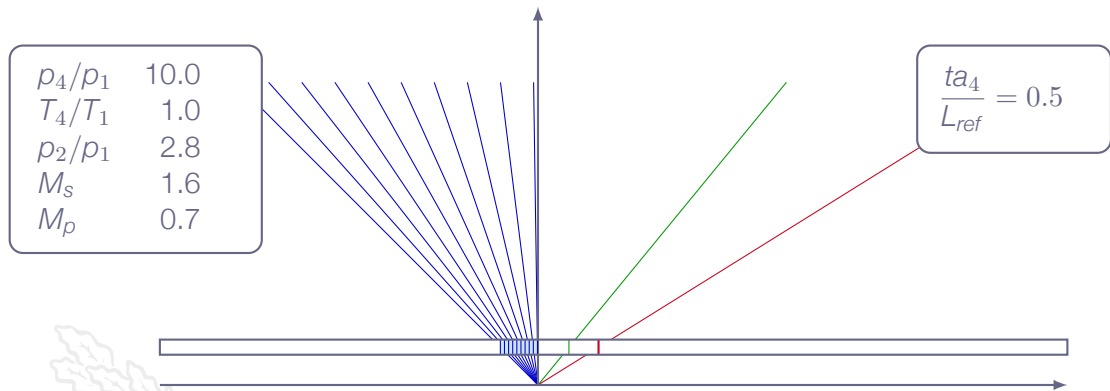
Along each C^- line u and a are **constants** which means that

$$\frac{dx}{dt} = u - a = \text{const}$$

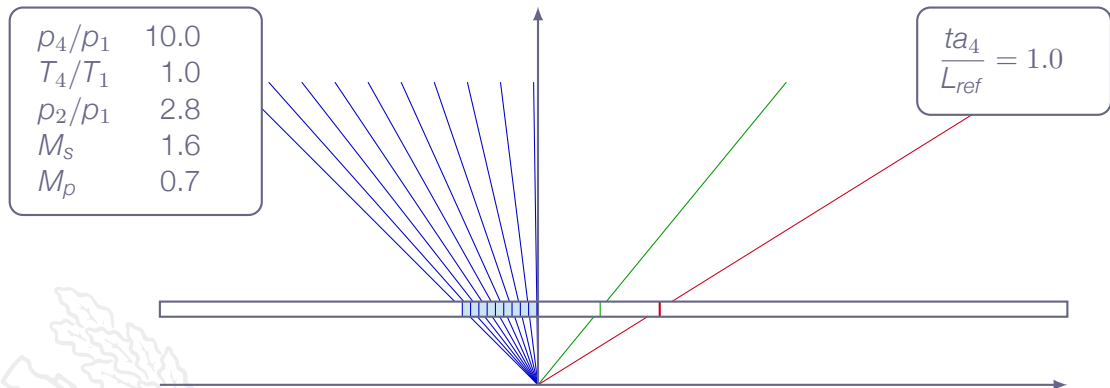
C^- characteristics are **straight lines** in xt -space



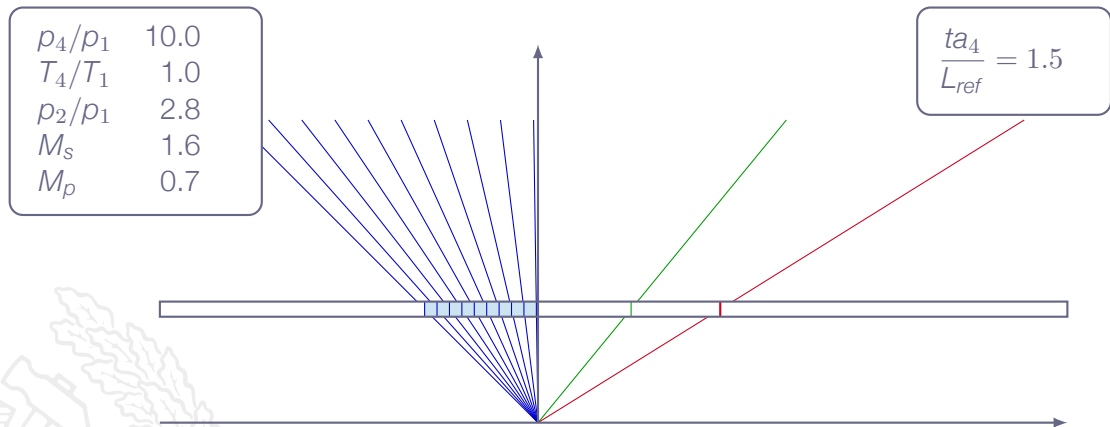
Expansion Waves - Shock Tube



Expansion Waves - Shock Tube



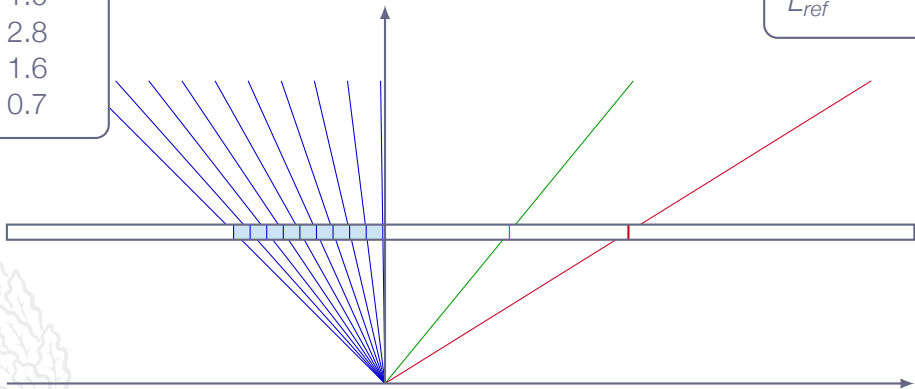
Expansion Waves - Shock Tube



Expansion Waves - Shock Tube

p_4/p_1	10.0
T_4/T_1	1.0
p_2/p_1	2.8
M_s	1.6
M_p	0.7

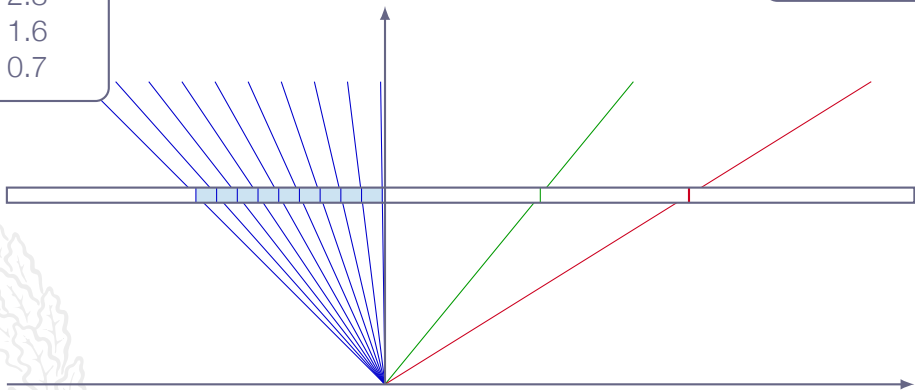
$$\frac{ta_4}{L_{ref}} = 2.0$$



Expansion Waves - Shock Tube

p_4/p_1	10.0
T_4/T_1	1.0
p_2/p_1	2.8
M_s	1.6
M_p	0.7

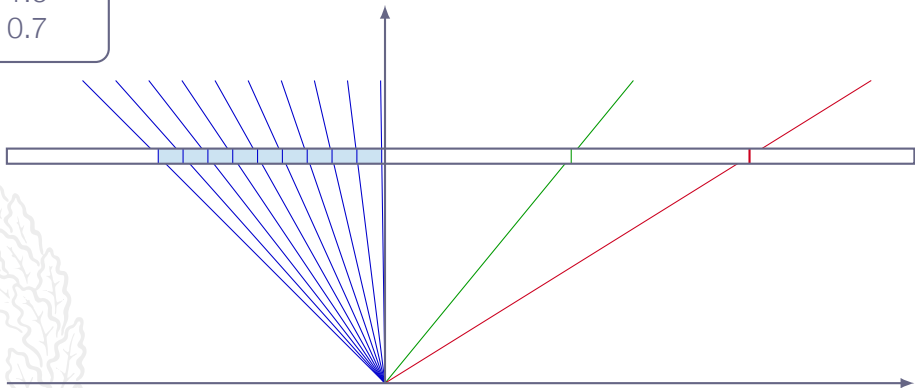
$$\frac{ta_4}{L_{ref}} = 2.5$$



Expansion Waves - Shock Tube

p_4/p_1	10.0
T_4/T_1	1.0
p_2/p_1	2.8
M_s	1.6
M_p	0.7

$$\frac{ta_4}{L_{ref}} = 3.0$$



Shock Tube Expansion Waves - Summary

The start and end conditions are the same for all C^+ lines

J^+ invariants have the same value for all C^+ characteristics

C^- characteristics are straight lines in xt -space

Simple expansion waves centered at $(x, t) = (0, 0)$

Expansion Waves

In a left-running expansion fan:

J^+ is constant throughout expansion fan, which implies:

$$u + \frac{2a}{\gamma - 1} = u_4 + \frac{2a_4}{\gamma - 1} = u_3 + \frac{2a_3}{\gamma - 1}$$

J^- is constant along C^- lines, but varies from one line to the next, which means that

$$u - \frac{2a}{\gamma - 1}$$

is constant along each C^- line

Expansion Waves

Since $u_4 = 0$ we obtain:

$$u + \frac{2a}{\gamma - 1} = u_4 + \frac{2a_4}{\gamma - 1} = \frac{2a_4}{\gamma - 1} \Rightarrow$$

$$\frac{a}{a_4} = 1 - \frac{1}{2}(\gamma - 1)\frac{u}{a_4}$$

with $a = \sqrt{\gamma RT}$ we get

$$\frac{T}{T_4} = \left[1 - \frac{1}{2}(\gamma - 1)\frac{u}{a_4} \right]^2$$

Expansion Wave Relations

Isentropic flow $\Rightarrow$ we can use the isentropic relations

complete description in terms of u/a_4

$$\frac{T}{T_4} = \left[1 - \frac{1}{2}(\gamma - 1) \frac{u}{a_4} \right]^2$$

$$\frac{\rho}{\rho_4} = \left[1 - \frac{1}{2}(\gamma - 1) \frac{u}{a_4} \right]^{\frac{2\gamma}{\gamma-1}}$$

$$\frac{\rho}{\rho_4} = \left[1 - \frac{1}{2}(\gamma - 1) \frac{u}{a_4} \right]^{\frac{2}{\gamma-1}}$$

Expansion Wave Relations

Since C^- characteristics are straight lines, we have:

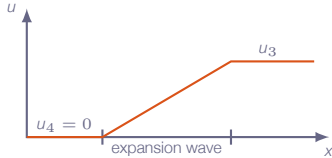
$$\frac{dx}{dt} = u - a \Rightarrow x = (u - a)t$$

$$\frac{a}{a_4} = 1 - \frac{1}{2}(\gamma - 1)\frac{u}{a_4} \Rightarrow a = a_4 - \frac{1}{2}(\gamma - 1)u \Rightarrow$$

$$x = \left[u - a_4 + \frac{1}{2}(\gamma - 1)u \right] t = \left[\frac{1}{2}(\gamma - 1)u - a_4 \right] t \Rightarrow$$

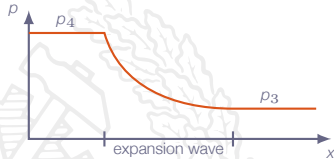
$$u = \frac{2}{\gamma + 1} \left[a_4 + \frac{x}{t} \right]$$

Expansion Wave Relations

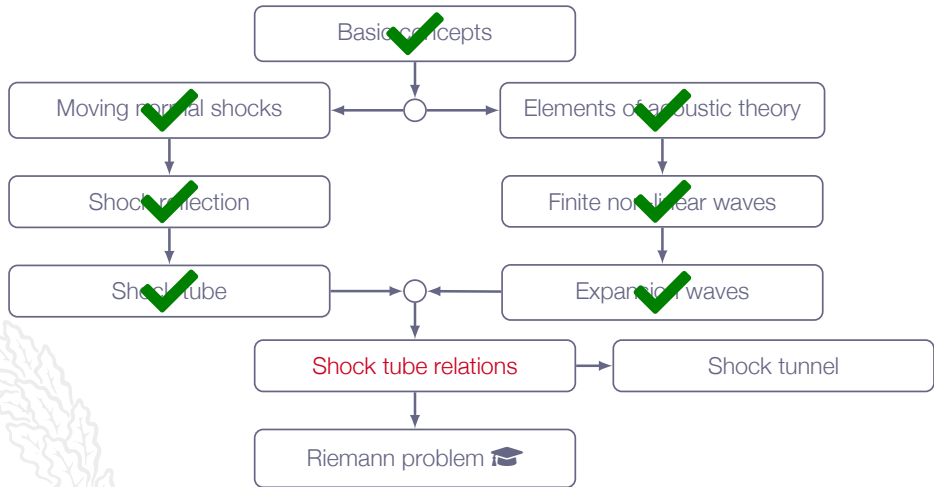


Expansion wave head is advancing to the left with speed a_4 into the stagnant gas

Expansion wave tail is advancing with speed $u_3 - a_3$, which may be positive or negative, depending on the initial states



Roadmap - Unsteady Wave Motion



Chapter 7.8

Shock Tube Relations



Shock Tube Relations

$$u_p = u_2 = \frac{a_1}{\gamma} \left(\frac{p_2}{p_1} - 1 \right) \left[\frac{\frac{2\gamma_1}{\gamma_1 + 1}}{\frac{p_2}{p_1} + \frac{\gamma_1 - 1}{\gamma_1 + 1}} \right]^{1/2}$$

$$\frac{p_3}{p_4} = \left[1 - \frac{\gamma_4 - 1}{2} \left(\frac{u_3}{a_4} \right) \right]^{2\gamma_4/(\gamma_4 - 1)}$$

solving for u_3 gives

$$u_3 = \frac{2a_4}{\gamma_4 - 1} \left[1 - \left(\frac{p_3}{p_4} \right)^{(\gamma_4 - 1)/(2\gamma_4)} \right]$$

Shock Tube Relations

But, $p_3 = p_2$ and $u_3 = u_2$ (no change in velocity and pressure over contact discontinuity)

$$\Rightarrow u_2 = \frac{2a_4}{\gamma_4 - 1} \left[1 - \left(\frac{p_2}{p_4} \right)^{(\gamma_4 - 1)/(2\gamma_4)} \right]$$

We have now two expressions for u_2 which gives us

$$\frac{a_1}{\gamma} \left(\frac{p_2}{p_1} - 1 \right) \left[\frac{\frac{2\gamma_1}{\gamma_1 + 1}}{\frac{p_2}{p_1} + \frac{\gamma_1 - 1}{\gamma_1 + 1}} \right]^{1/2} = \frac{2a_4}{\gamma_4 - 1} \left[1 - \left(\frac{p_2}{p_4} \right)^{(\gamma_4 - 1)/(2\gamma_4)} \right]$$

Shock Tube Relations

Rearranging gives:

$$\frac{p_4}{p_1} = \frac{p_2}{p_1} \left\{ 1 - \frac{(\gamma_4 - 1)(a_1/a_4)(p_2/p_1 - 1)}{\sqrt{2\gamma_1 [2\gamma_1 + (\gamma_1 + 1)(p_2/p_1 - 1)]}} \right\}^{-2\gamma_4/(\gamma_4 - 1)}$$

p_2/p_1 as implicit function of p_4/p_1

for a given p_4/p_1 , p_2/p_1 will increase with decreased a_1/a_4

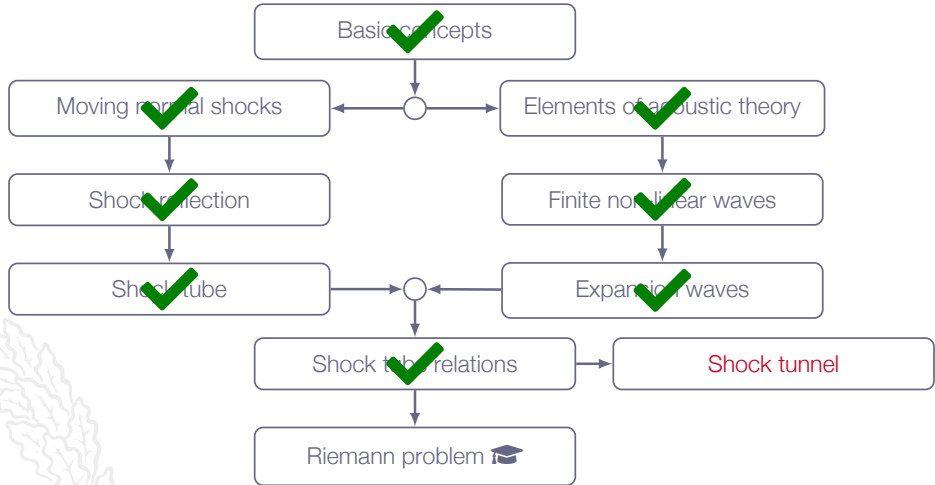
$$a = \sqrt{\gamma RT} = \sqrt{\gamma(R_u/M)T}$$

the speed of sound in a light gas is higher than in a heavy gas

driver gas: low molecular weight, high temperature

driven gas: high molecular weight, low temperature

Roadmap - Unsteady Wave Motion



Shock Tunnel

Addition of a convergent-divergent nozzle to a shock tube configuration

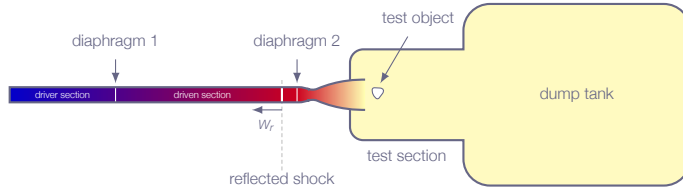
Capable of producing flow conditions which are close to those during the reentry of a space vehicles into the earth's atmosphere

high-enthalpy, hypersonic flows (short time)
real gas effects

Example - Aachen TH2:

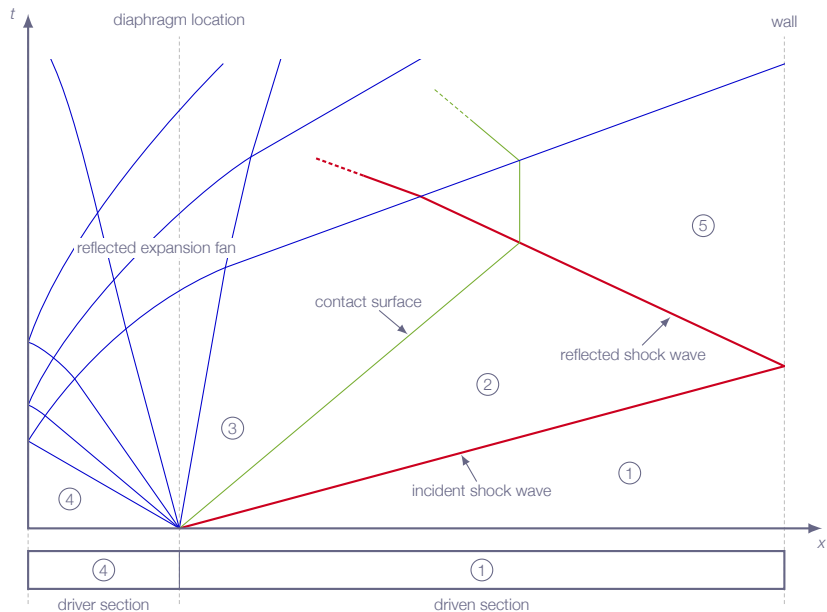
velocities up to 4 km/s
stagnation temperatures of several thousand degrees

Shock Tunnel



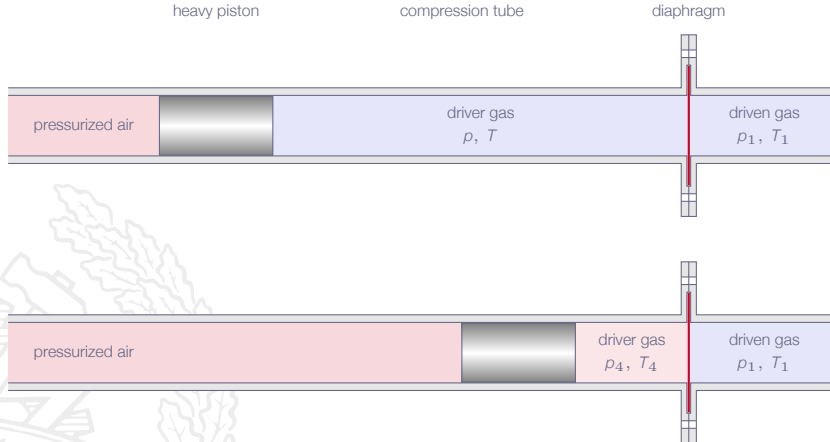
1. High pressure in region 4 (driver section)
diaphragm 1 burst
primary shock generated
2. Primary shock reaches end of shock tube
shock reflection
3. High pressure in region 5
diaphragm 2 burst
nozzle flow initiated
hypersonic flow in test section

Shock Tunnel



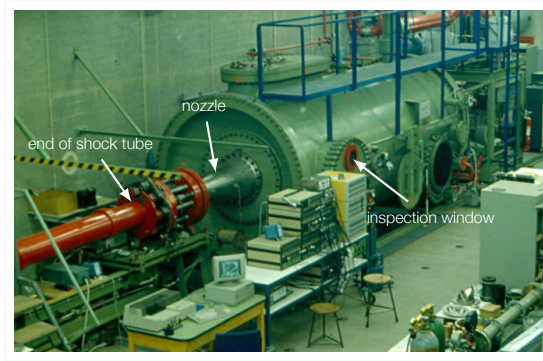
Shock Tunnel

By adding a compression tube to the shock tube a very high p_4 and T_4 may be achieved for any gas in a fairly simple manner



The Aachen Shock Tunnel - TH2

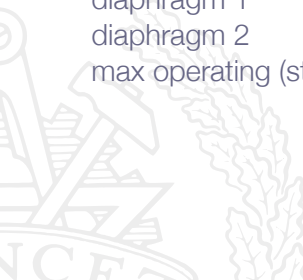
Shock tunnel built 1975



The Aachen Shock Tunnel - TH2

Shock tube specifications:

diameter	140 mm
driver section	6.0 m
driven section	15.4 m
diaphragm 1	10 mm stainless steel
diaphragm 2	copper/brass sheet
max operating (steady) pressure	1500 bar



The Aachen Shock Tunnel - TH2

Driver gas (usually helium):

$$100 \text{ bar} < p_4 < 1500 \text{ bar}$$

electrical preheating (optional) to 600 K

Driven gas:

$$0.1 \text{ bar} < p_1 < 10 \text{ bar}$$

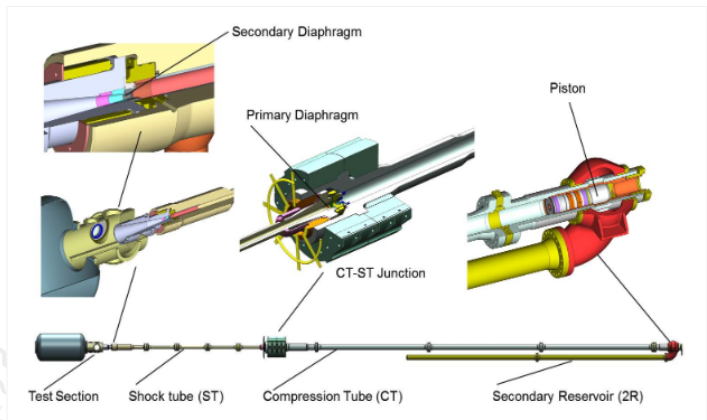
Dump tank evacuated before test

The Aachen Shock Tunnel - TH2

initial conditions			shock		reservoir		free stream			
p_4 [bar]	T_4 [K]	p_1 [bar]	M_s	p_2 [bar]	p_5 [bar]	T_5 [K]	M_∞	T_∞ [K]	u_∞ [m/s]	p_∞ [mbar]
100	293	1.0	3.3	12	65	1500	7.7	125	1740	7.6
370	500	1.0	4.6	26	175	2500	7.4	250	2350	20.0
720	500	0.7	5.6	50	325	3650	6.8	460	3910	42.0
1200	500	0.6	6.8	50	560	4600	6.5	700	3400	73.0
100	293	0.9	3.4	12	65	1500	11.3	60	1780	0.6
450	500	1.2	4.9	29	225	2700	11.3	120	2480	1.5
1300	520	0.7	6.4	46	630	4600	12.1	220	3560	1.2
26	293	0.2	3.4	12	15	1500	11.4	60	1780	0.1
480	500	0.2	6.6	50	210	4600	11.0	270	3630	0.7
100	293	1.0	3.4	12	65	1500	7.7	130	1750	7.3
370	500	1.0	5.1	27	220	2700	7.3	280	2440	26.3

The Caltech Shock Tunnel - T5

Free-piston shock tunnel



The Caltech Shock Tunnel - T5

Compression tube (CT):

length 30 m, diameter 300 mm
free piston (120 kg)
max piston velocity: 300 m/s
driven by compressed air (80 bar - 150 bar)

Shock tube (ST):

length 12 m, diameter 90 mm
driver gas: helium + argon
driven gas: air
diaphragm 1: 7 mm stainless steel
 p_4 max 1300 bar

The Caltech Shock Tunnel - T5

Reservoir conditions:

$$p_5 \text{ 1000 bar}$$

$$T_5 \text{ 10000 K}$$

Freestream conditions (design conditions):

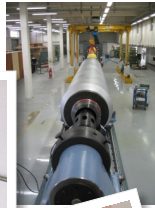
$$M_\infty \text{ 5.2}$$

$$T_\infty \text{ 2000 K}$$

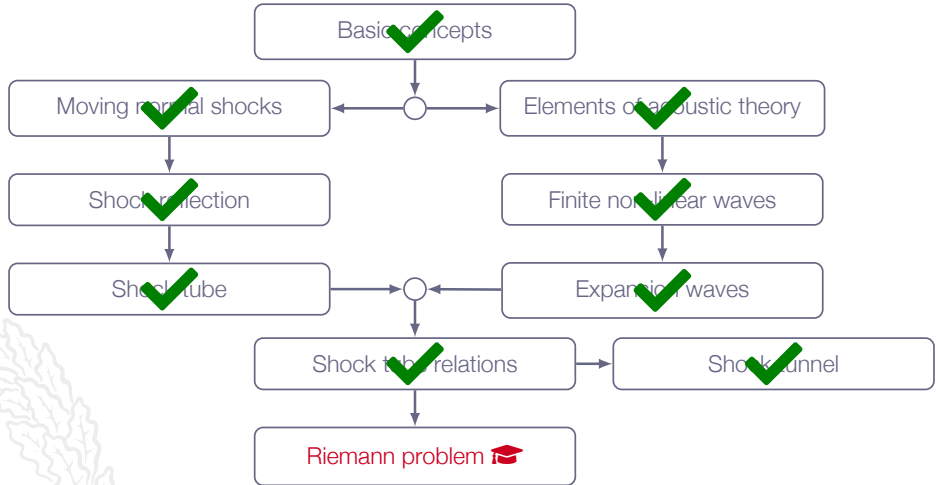
$$p_\infty \text{ 0.3 bar}$$

typical test time 1 ms

Other Examples of Shock Tunnels



Roadmap - Unsteady Wave Motion





The shock tube problem is a special case of the general **Riemann Problem**

"... A Riemann problem, named after Bernhard Riemann, consists of an initial value problem composed by a conservation equation together with piece-wise constant data having a single discontinuity ..."

Wikipedia





May show that solutions to the shock tube problem have the general form:

$$p = p(x/t)$$

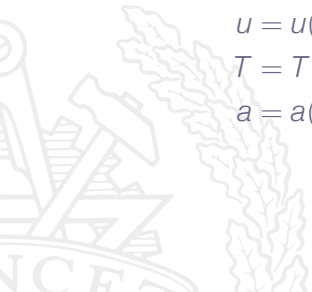
$$\rho = \rho(x/t)$$

$$u = u(x/t)$$

$$T = T(x/t)$$

$$a = a(x/t)$$

where $x = 0$ denotes the position of the initial jump between states 1 and 4





Numerical method:

Finite-Volume Method (FVM) solver

three-stage Runge-Kutta time stepping

third-order characteristic upwinding
scheme

local artificial damping

Left side conditions (state 4):

$$\rho = 2.4 \text{ kg/m}^3$$

$$u = 0.0 \text{ m/s}$$

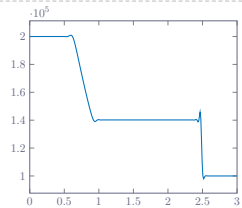
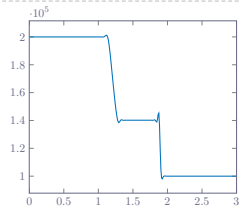
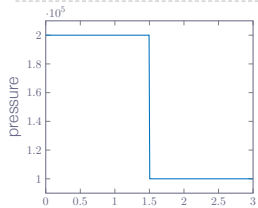
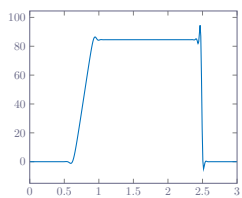
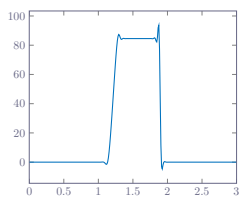
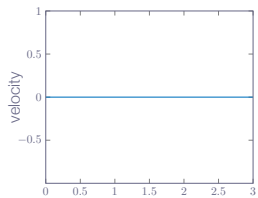
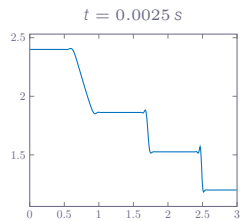
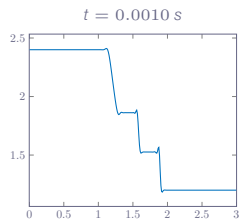
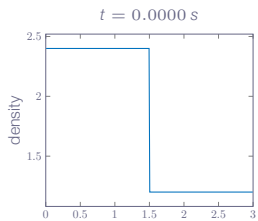
$$p = 2.0 \text{ bar}$$

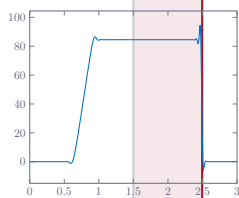
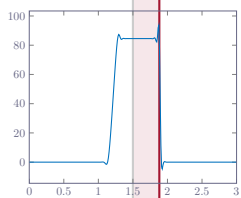
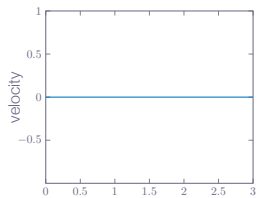
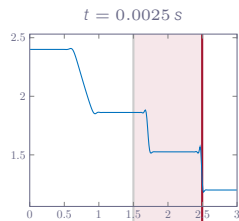
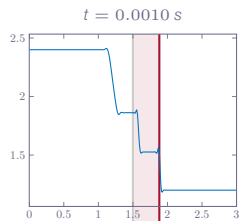
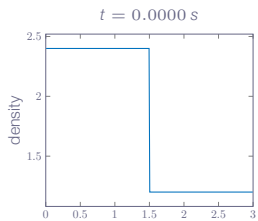
Right side conditions (state 1):

$$\rho = 1.2 \text{ kg/m}^3$$

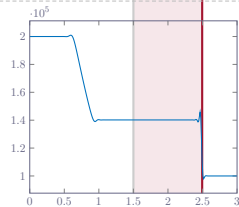
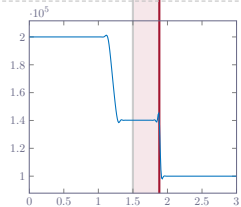
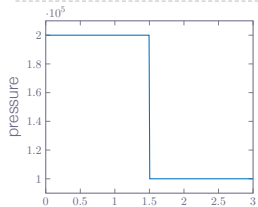
$$u = 0.0 \text{ m/s}$$

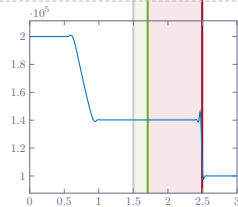
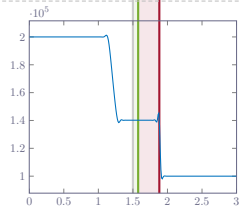
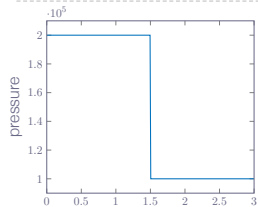
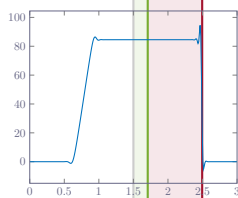
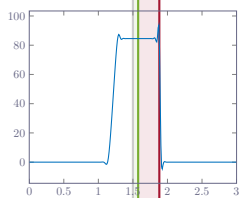
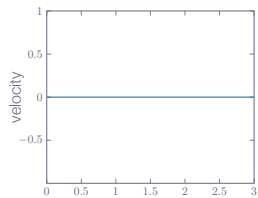
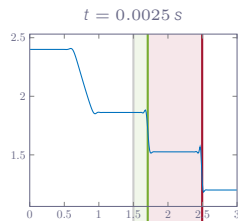
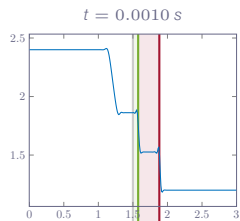
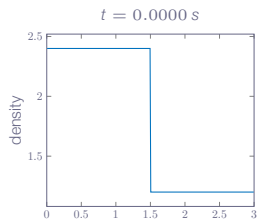
$$p = 1.0 \text{ bar}$$



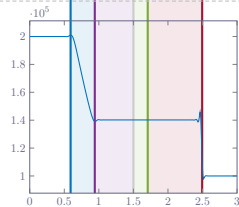
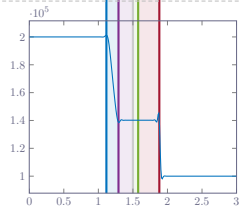
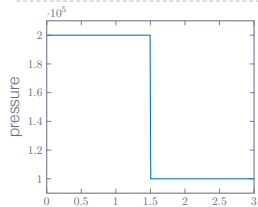
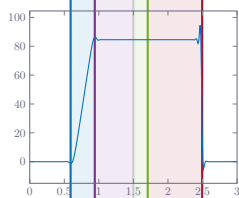
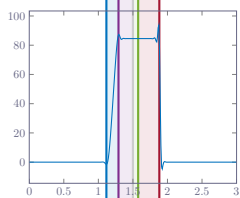
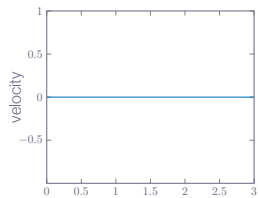
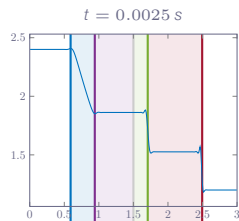
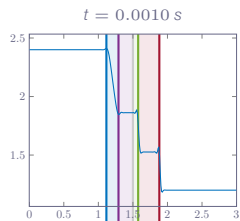
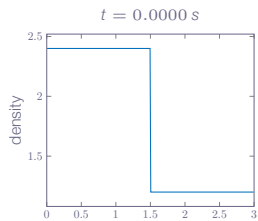


incident shock



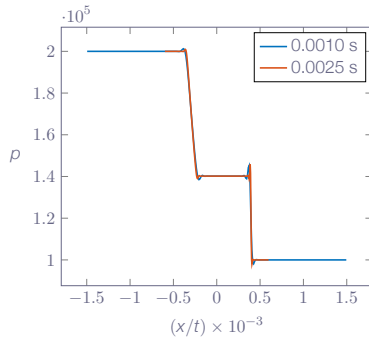
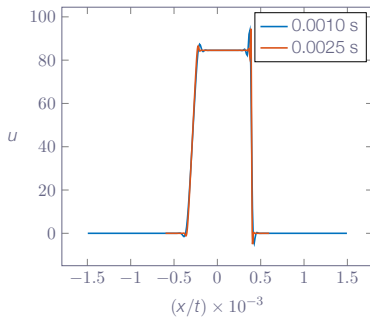
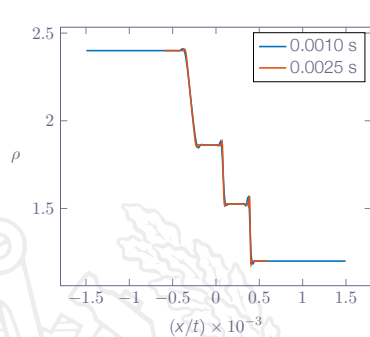


incident shock
contact discontinuity



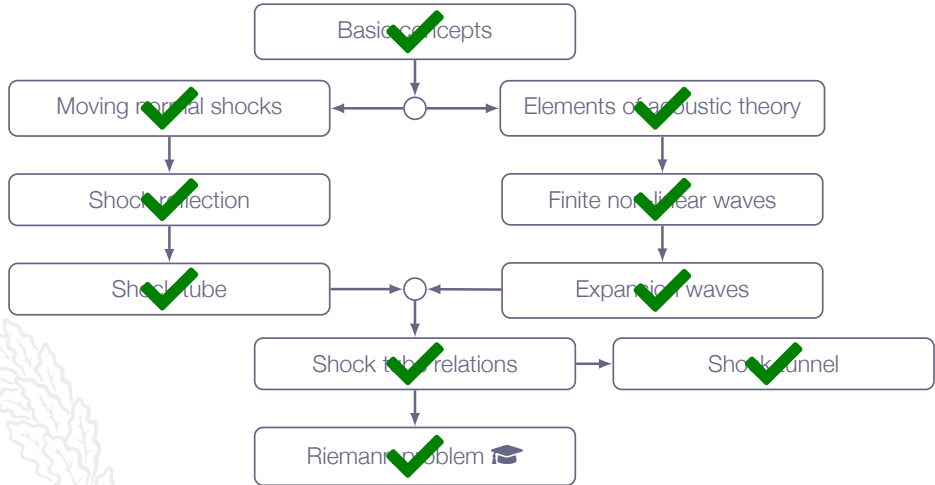
incident shock
contact discontinuity
expansion wave

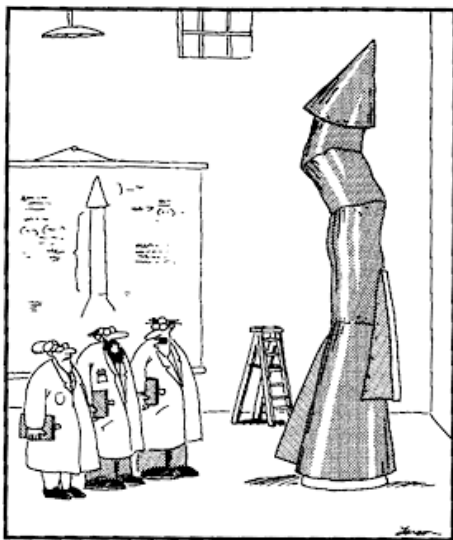
Riemann Problem - Shock Tube Simulation



The solution can be made self similar by plotting the flow field variables as function of x/t

Roadmap - Unsteady Wave Motion





"It's time we face reality, my friend. ... We're not exactly rocket scientists."