Compressible Flow - TME085

Lecture 8

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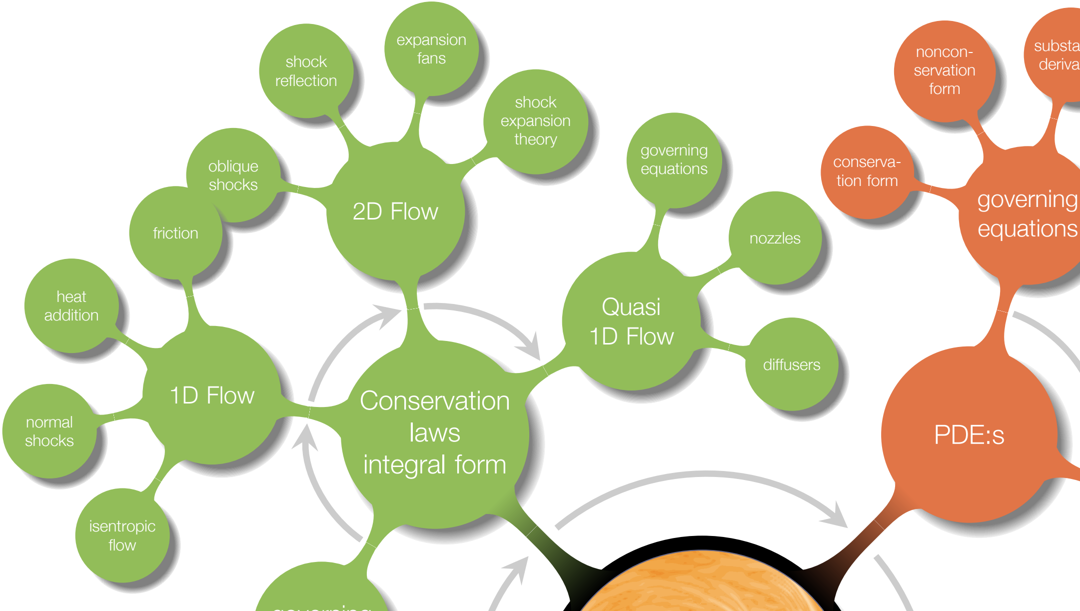
Chapter 5 - Quasi-One-Dimensional Flow

Overview



shock reflection
expansion fans
shock expansion theory
oblique shocks
friction
2D Flow
governing equations
nozzles
Quasi 1D Flow
diffusers
heat addition
1D Flow
normal shocks
isentropic flow
Conservation laws integral form
noncon-servation form
conserva-tion form
governing equations
PDE:s

Learning Outcomes

4 **Present** at least two different formulations of the governing equations for compressible flows and **explain** what basic conservation principles they are based on

6 **Define** the special cases of calorically perfect gas, thermally perfect gas and real gas and **explain** the implication of each of these special cases

7 **Explain** why entropy is important for flow discontinuities

8 **Derive** (marked) and **apply** (all) of the presented mathematical formulae for classical gas dynamics

  a 1D isentropic flow*

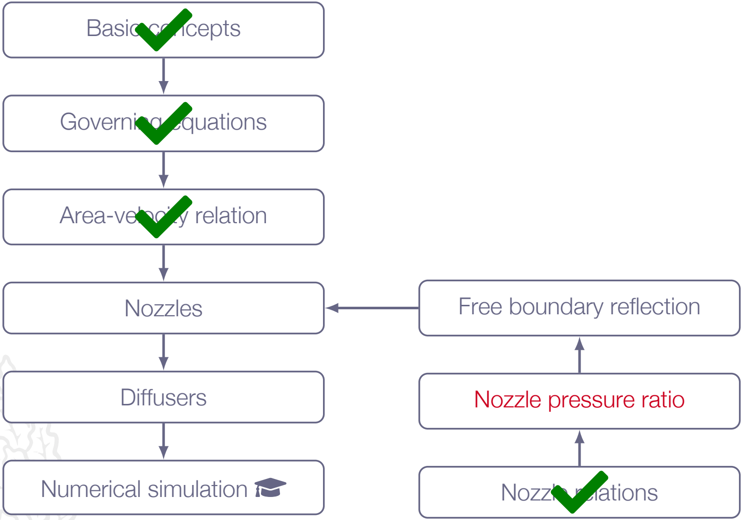
  b normal shocks*

  i detached blunt body shocks, nozzle flows

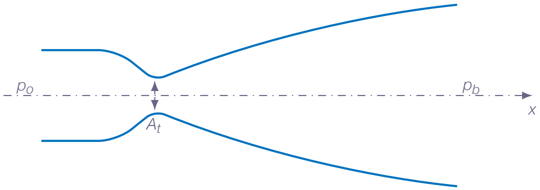
9 **Solve** engineering problems involving the above-mentioned phenomena (8a-8k)

what does quasi-1D mean? either the flow is 1D or not, or?

Roadmap - Quasi-One-Dimensional Flow

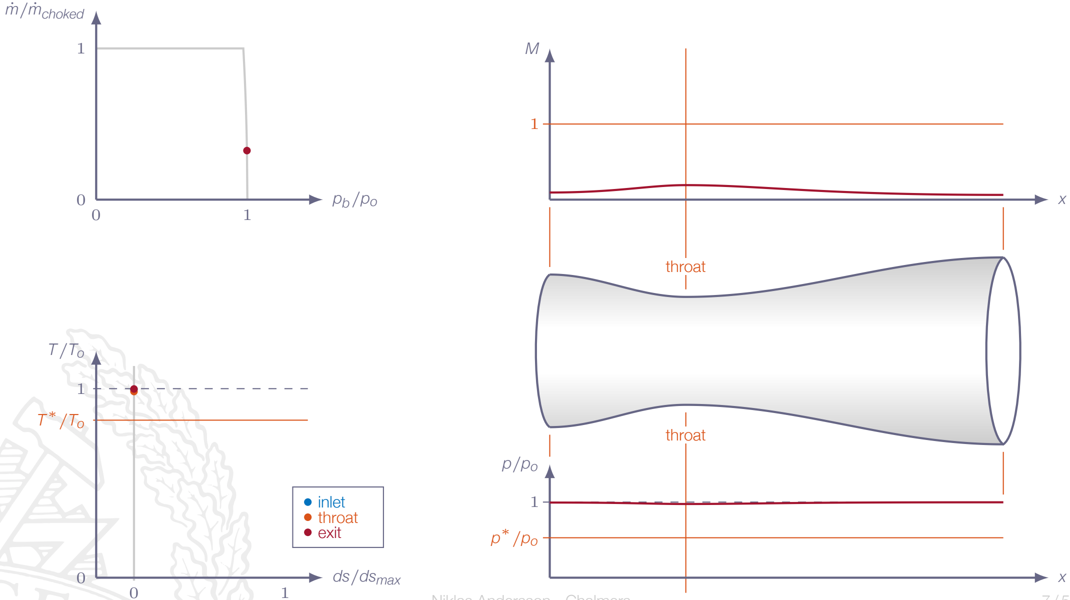


Nozzle Flow with Varying Pressure Ratio



$A(x)$	area function
A_t	$\min\{A(x)\}$
p_o	inlet total pressure
p_b	outlet static pressure (ambient pressure)
p_o/p_b	pressure ratio

Nozzle Flow with Varying Pressure Ratio



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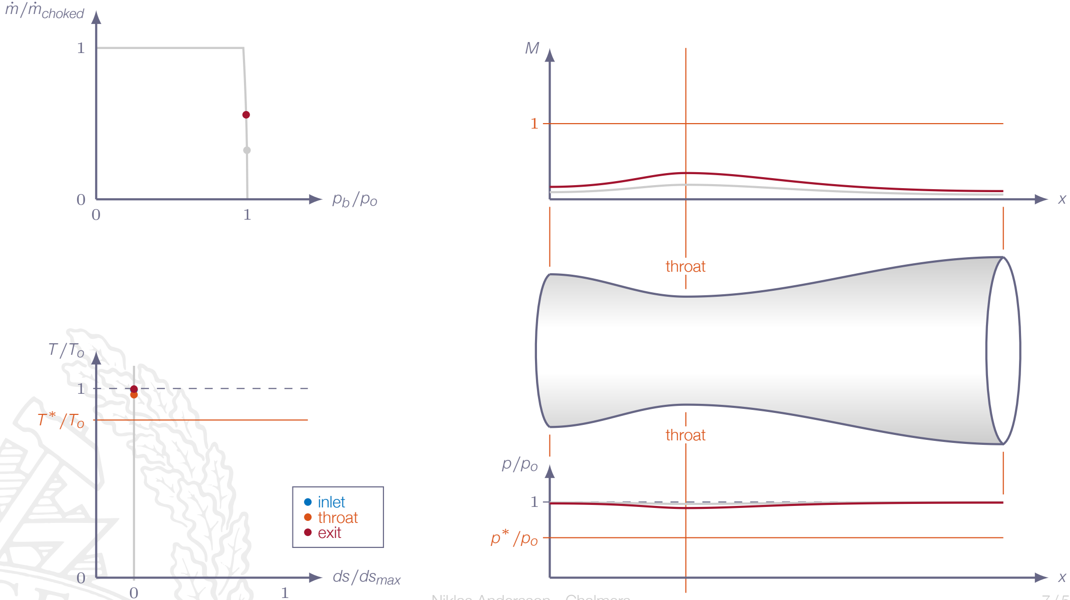
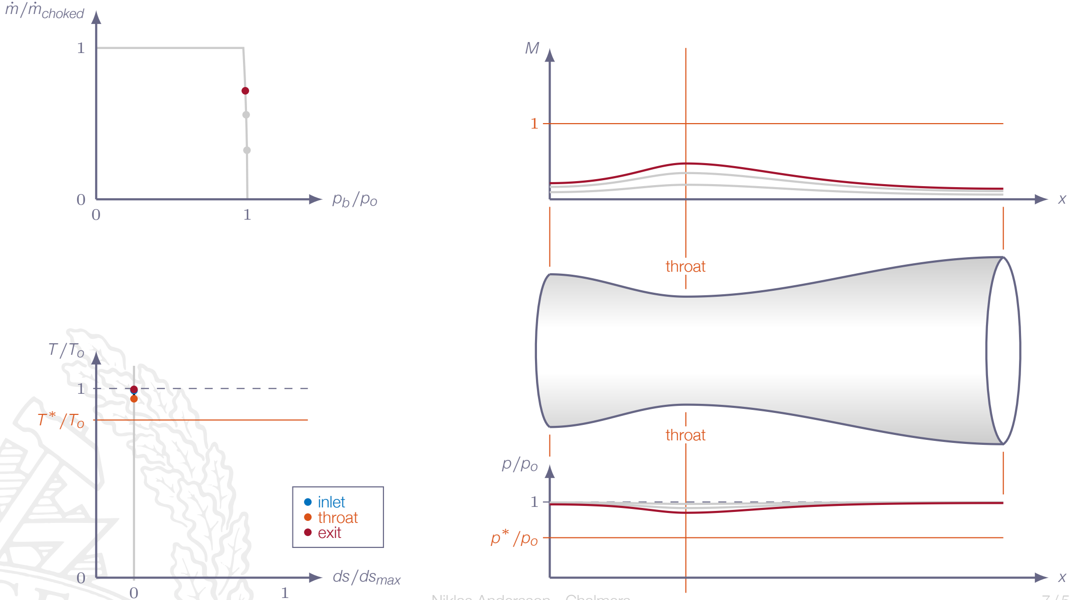
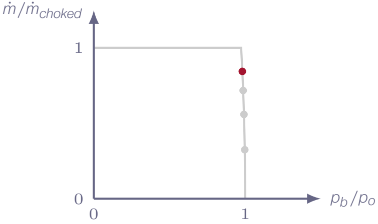
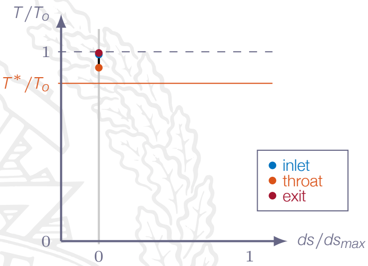


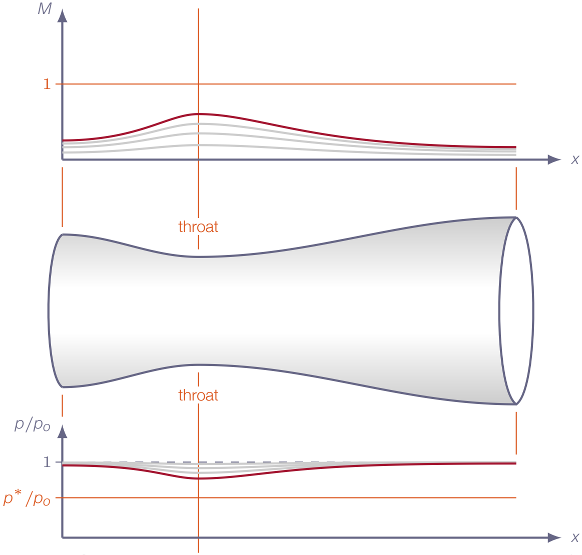
Nozzle Flow with Varying Pressure Ratio



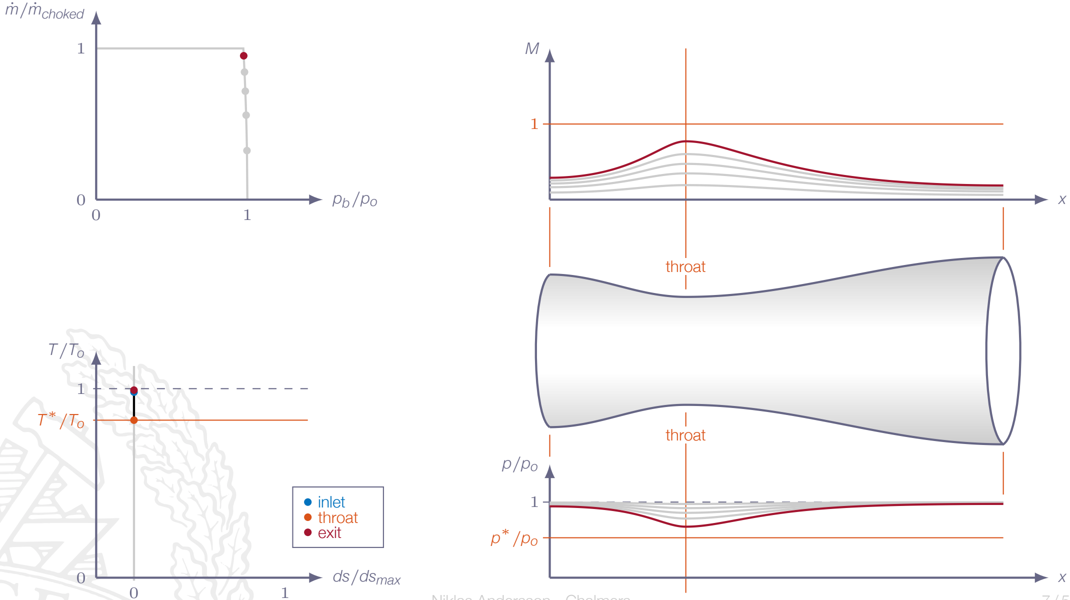
Nozzle Flow with Varying Pressure Ratio



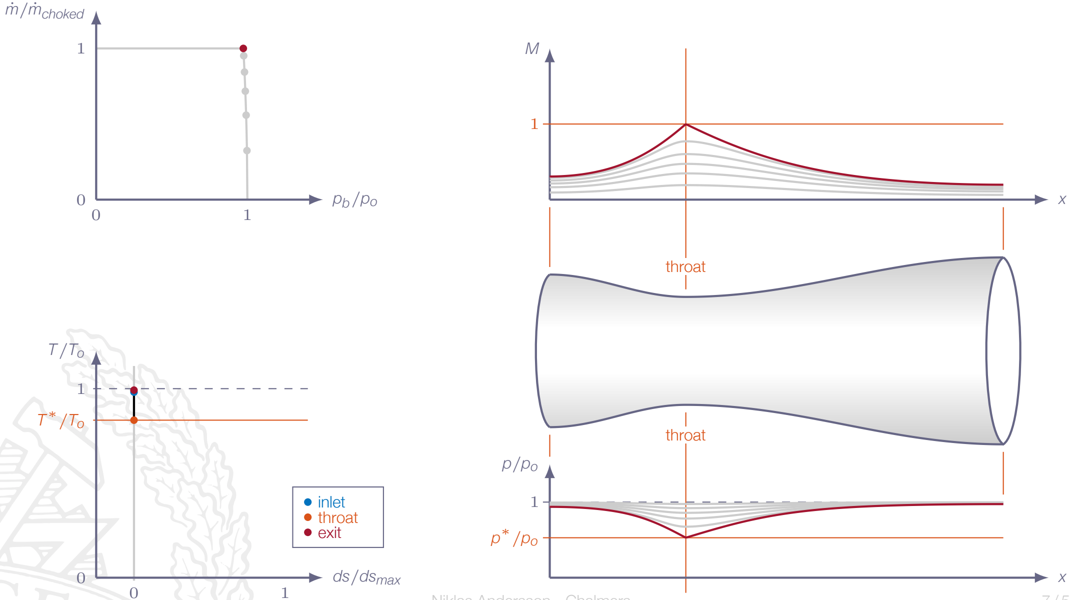




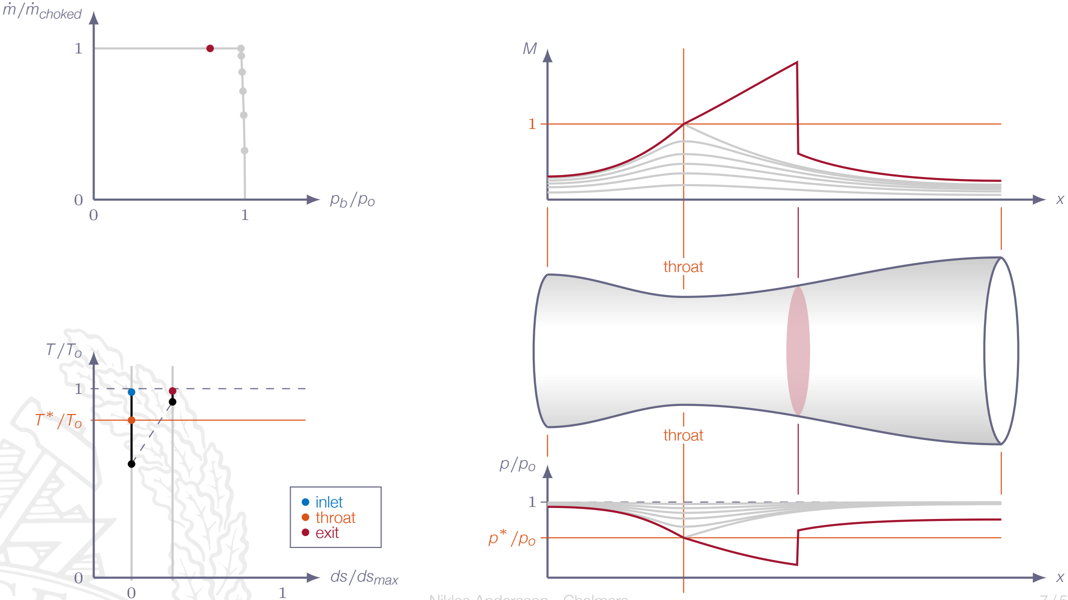
Nozzle Flow with Varying Pressure Ratio



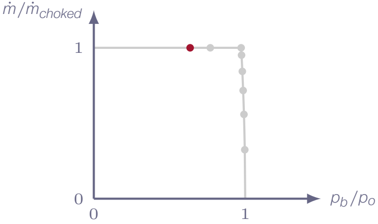
Nozzle Flow with Varying Pressure Ratio

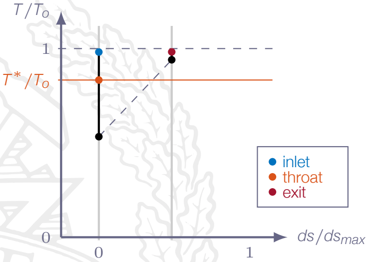


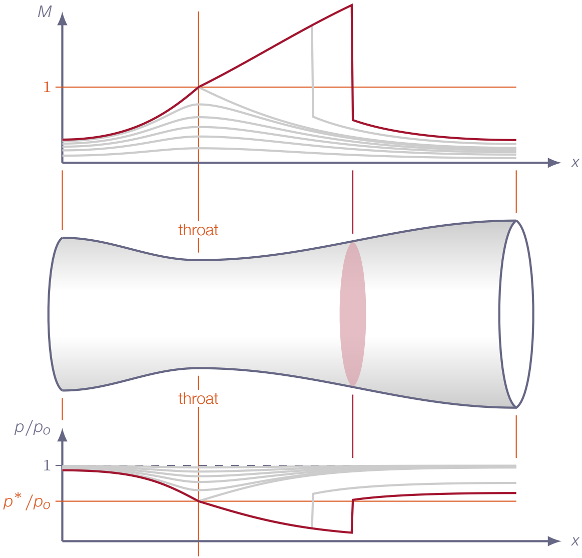
Nozzle Flow with Varying Pressure Ratio



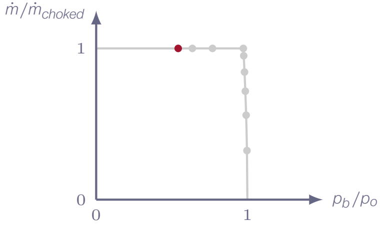
Nozzle Flow with Varying Pressure Ratio

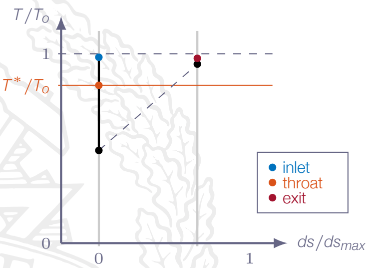


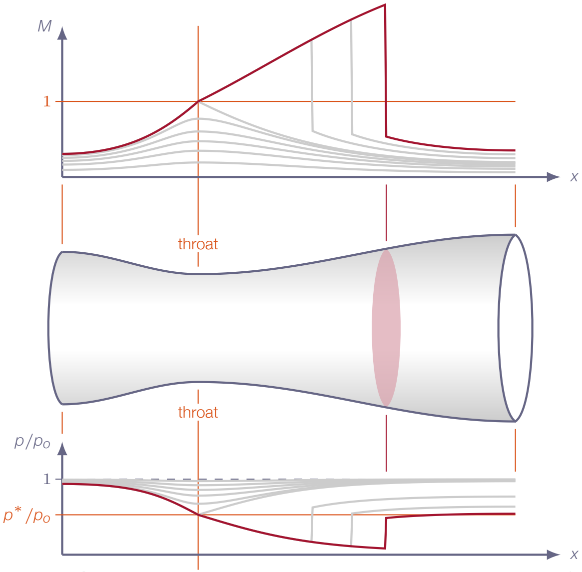




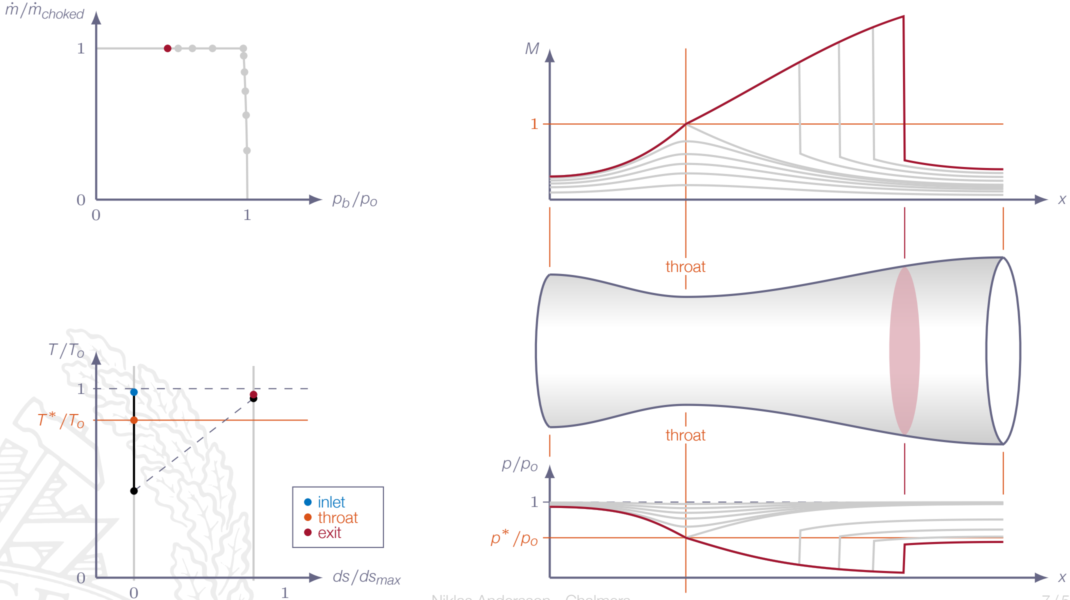
Nozzle Flow with Varying Pressure Ratio



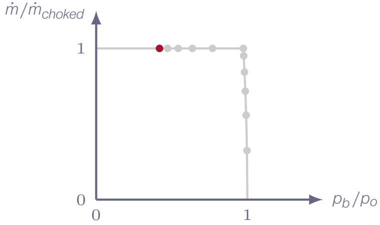


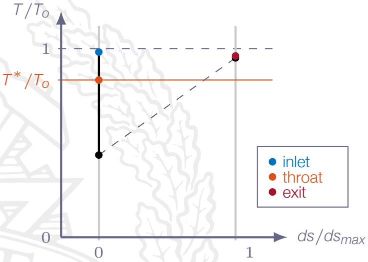


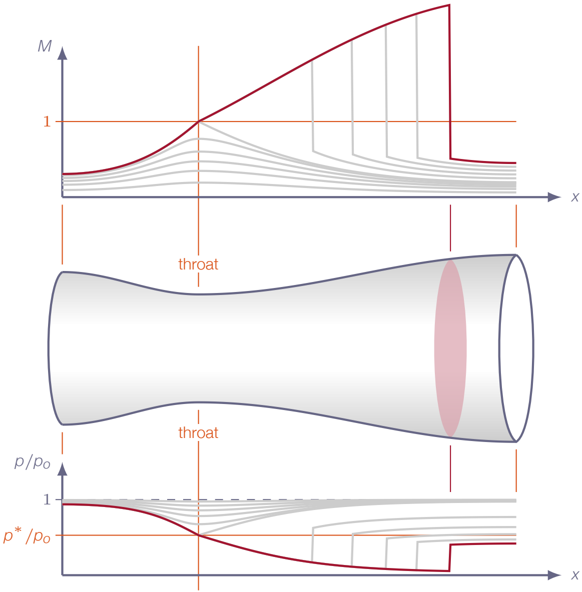
Nozzle Flow with Varying Pressure Ratio



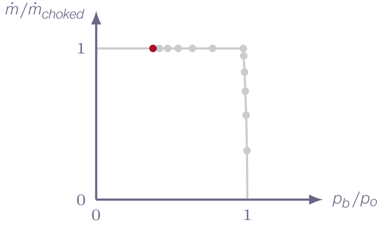
Nozzle Flow with Varying Pressure Ratio

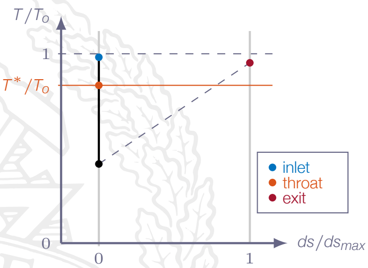


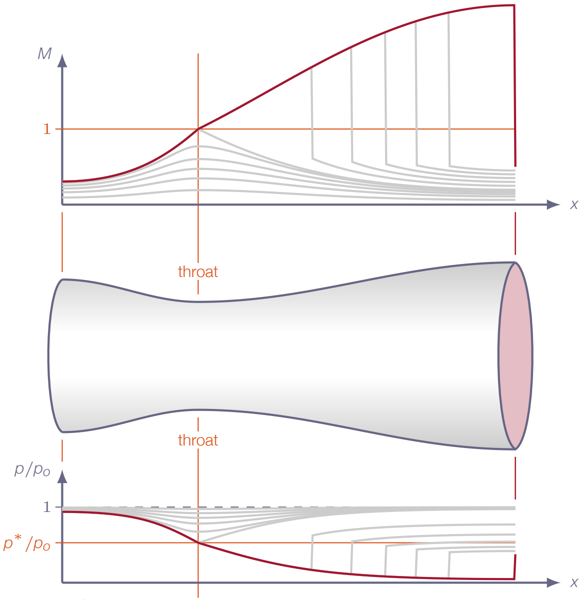




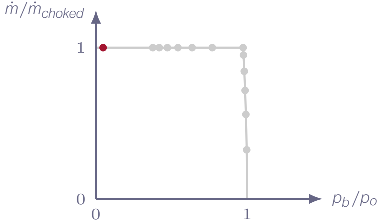
Nozzle Flow with Varying Pressure Ratio

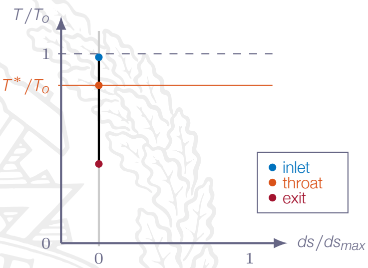


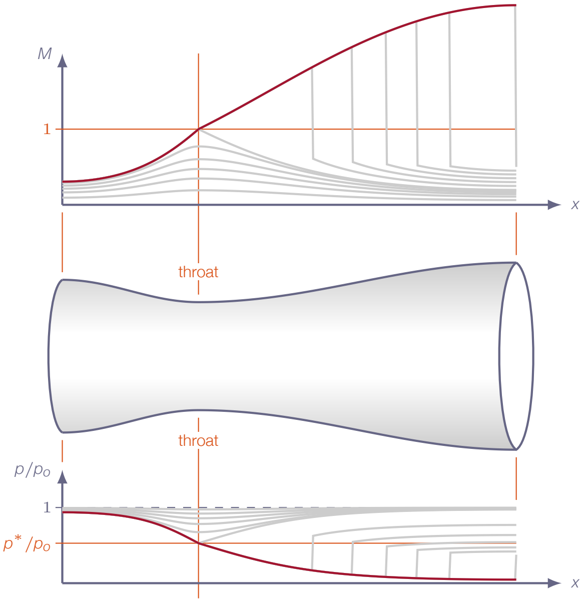




Nozzle Flow with Varying Pressure Ratio



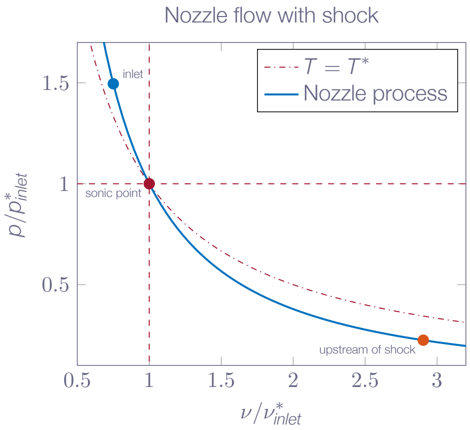




Nozzle Flow with Internal Shock

The nozzle flow process follows an isentrope up to the location of the internal normal shock

Sonic conditions at the nozzle throat

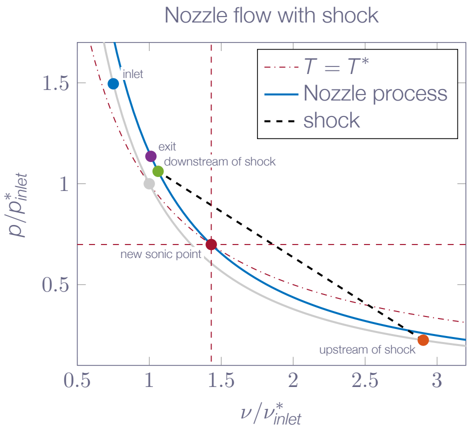


Nozzle Flow with Internal Shock

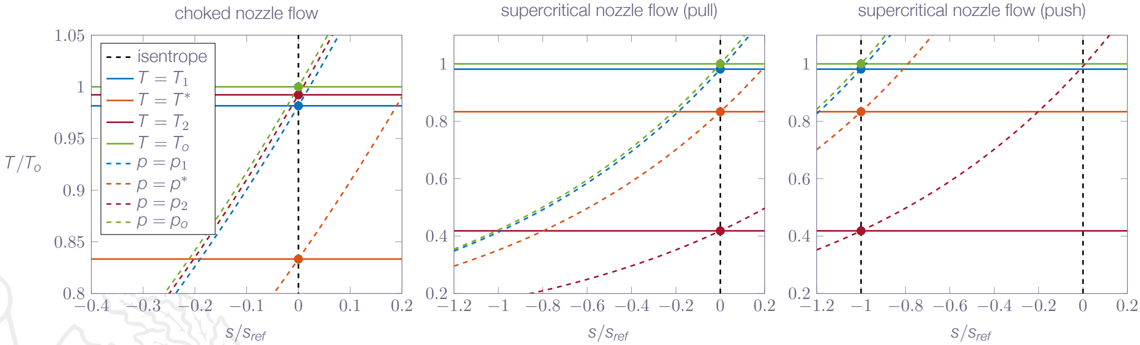
The normal shock moves the process line to another isentrope

T_o and thus T^* is not affected by the shock

p_o decreases over the shock which means that p^* decreases and ν^* increases



Nozzle Operation - Pull vs. Push

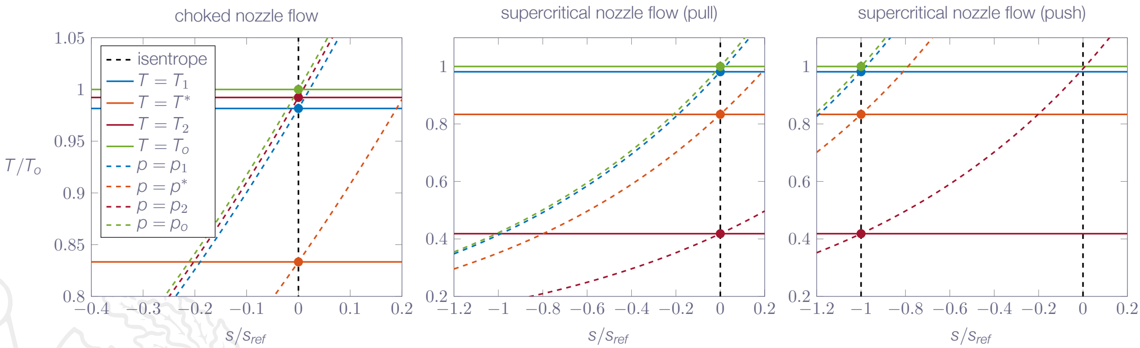


Nozzle Pressure Ratio $NPR = p_o/p_b$

Pull - increase NPR by reducing the back pressure (p_b)

Push - increase NPR by increasing the inlet total pressure (p_o)

Nozzle Operation - Pull vs. Push

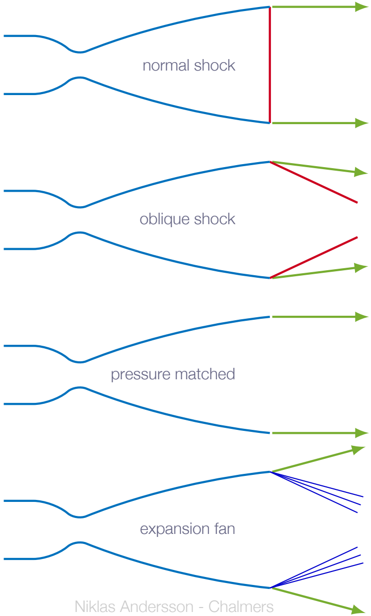


$$\left.\begin{array}{l} p^*_{push} > p^*_{pull} \\ T^*_{push} = T^*_{pull} \end{array}\right\} \Rightarrow \left\{\begin{array}{l} \rho^*_{push} > \rho^*_{pull} \\ a^*_{push} = a^*_{pull} \end{array}\right.$$

$$\dot{m} = \rho^* a^* A^* \;(A^*_{push} = A^*_{pull})$$

$$\dot{m}_{push} > \dot{m}_{pull} = \dot{m}_{choked}$$

Nozzle Flow with Varying Pressure Ratio - Downstream Flow



$p_o/p_b = (p_o/p_b)_{ne}$
normal shock at nozzle exit

$(p_o/p_b)_{ne} < p_o/p_b < (p_o/p_b)_{sc}$
overexpanded nozzle flow

$p_o/p_b = (p_o/p_b)_{sc}$
pressure matched nozzle flow

$p_o/p_b > (p_o/p_b)_{sc}$
underexpanded nozzle flow

Nozzle Flow with Varying Pressure Ratio (Summary)

$(p_o/p_b) < (p_o/p_b)_{cr}$

- **subsonic, isentropic** flow throughout the nozzle
- the mass flow changes with p_b, *i.e.* the flow is not choked

$(p_o/p_b) = (p_o/p_b)_{cr}$

- **sonic** flow ($M = 1.0$) at the throat
- the flow will flip to the supersonic solution downstream of the throat, for an infinitesimal increase of (p_o/p_b)

$(p_o/p_b)_{cr} < (p_o/p_b) < (p_o/p_b)_{ne}$

- the flow is **choked** (fixed mass flow)
- a **normal shock** will appear downstream of the throat, with strength and position depending on (p_o/p_b)

Nozzle Flow with Varying Pressure Ratio (Summary)

$(p_o/p_b) = (p_o/p_b)_{ne}$

- **normal shock** at the nozzle exit
- **supersonic, isentropic** flow from throat to exit

$(p_o/p_b)_{ne} < (p_o/p_b) < (p_o/p_b)_{sc}$

- **overexpanded** flow (supersonic, isentropic flow from throat to exit)
- **oblique shocks** formed downstream of the nozzle exit

$(p_o/p_b) = (p_o/p_b)_{sc}$

- **supercritical** flow (pressure matched)
- supersonic, isentropic flow from the throat and downstream of the nozzle exit

$(p_o/p_b)_{sc} < (p_o/p_b)$

- **underexpanded** flow (supersonic, isentropic flow from throat to exit)
- **expansion fans** formed downstream of the nozzle exit

Nozzle Flow with Varying Pressure Ratio - Q1D Limitations

Quasi-one-dimensional theory

When the interior normal shock is ”pushed out” through the exit (by increasing (p_o/p_b), *i.e.* lowering the back pressure), it disappears completely.

The flow through the nozzle is then **shock free** (and thus also **isentropic** since we neglect viscosity).

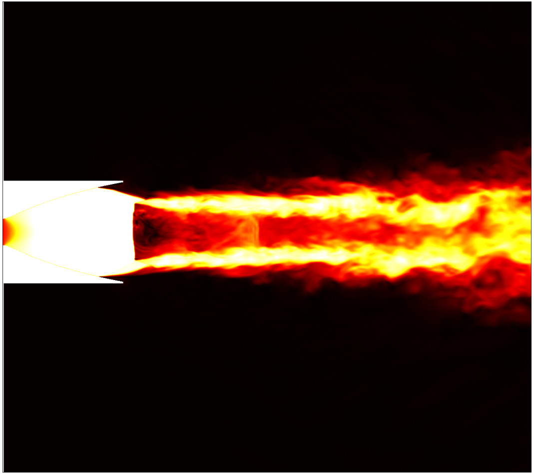
Three-dimensional nozzle flow

When the interior normal shock is ”pushed out” through the exit (by increasing (p_o/p_b)), an **oblique shock** is formed outside of the nozzle exit.

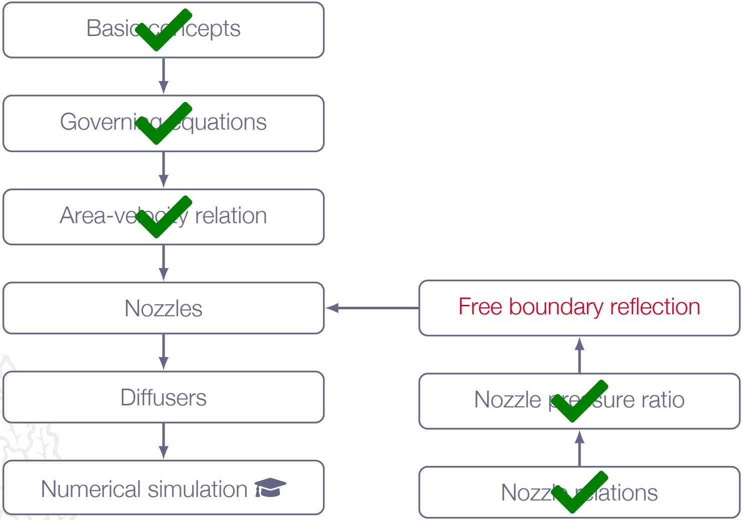
For the exact **supercritical** value of (p_o/p_b) this oblique shock disappears.

For (p_o/p_b) above the supercritical value an **expansion fan** is formed at the nozzle exit.

3D Simulations of Nozzle Flow



Roadmap - Quasi-One-Dimensional Flow

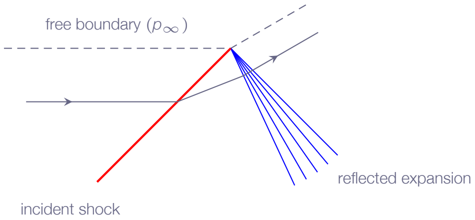


Chapter 5.6
Wave Reflection From a Free Boundary

Free-Boundary Reflection

Free boundary - shear layer, interface between different fluids, etc

Free-Boundary Reflection - Shock Reflection

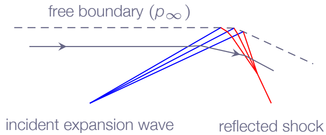


No discontinuity in pressure at the free boundary possible

Incident **shock reflects as expansion** waves at the free boundary

Reflection results in **net turning** of the flow

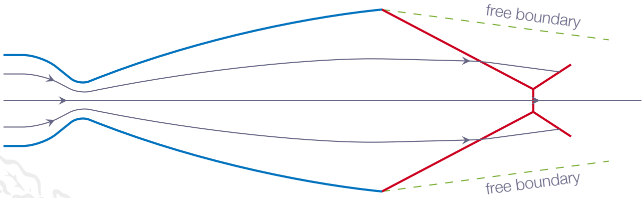
Free-Boundary Reflection - Expansion Wave Reflection



- No discontinuity in pressure at the free boundary possible
- Incident **expansion** waves **reflects as compression** waves at the free boundary
- Finite compression waves coalesces into a shock
- Reflection results in **net turning** of the flow

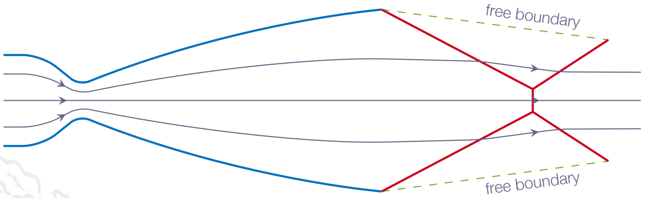
Free-Boundary Reflection - System of Reflections

overexpanded nozzle flow



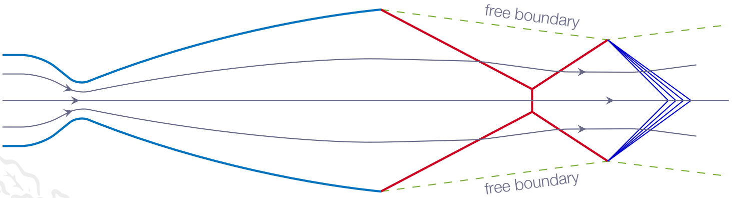
Free-Boundary Reflection - System of Reflections

shock reflection at jet centerline



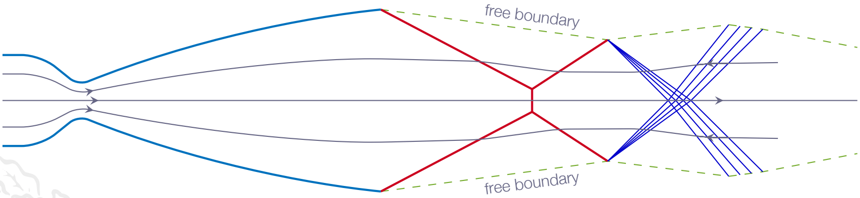
Free-Boundary Reflection - System of Reflections

shock reflection at free boundary



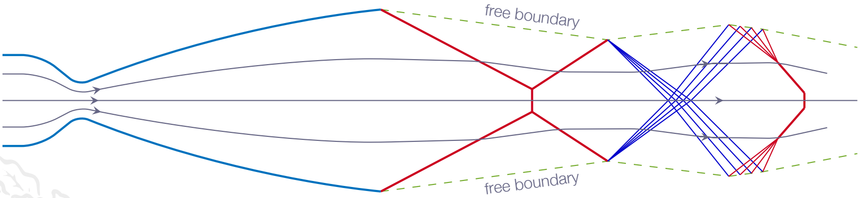
Free-Boundary Reflection - System of Reflections

expansion wave reflection at jet centerline



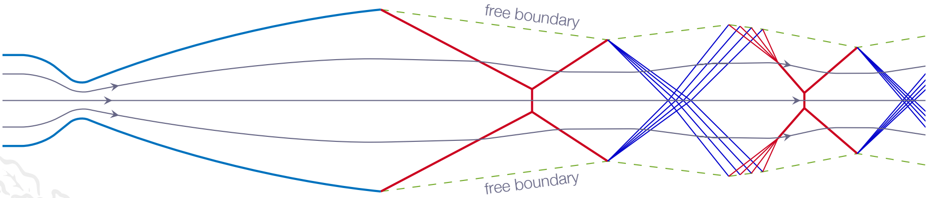
Free-Boundary Reflection - System of Reflections

expansion wave reflection at free boundary



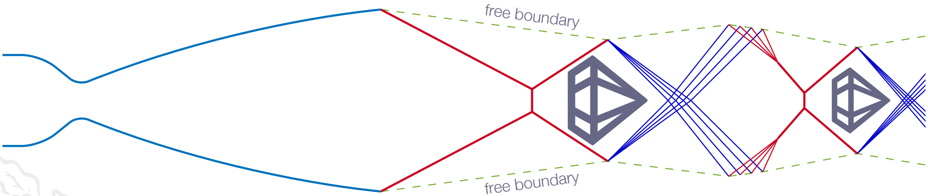
Free-Boundary Reflection - System of Reflections

repeated shock/expansion system



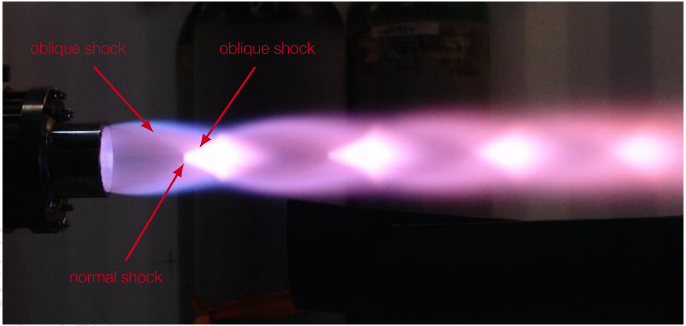
Free-Boundary Reflection - System of Reflections

shock diamonds



Free-Boundary Reflection - System of Reflections

overexpanded jet



Free-Boundary Reflection - Summary

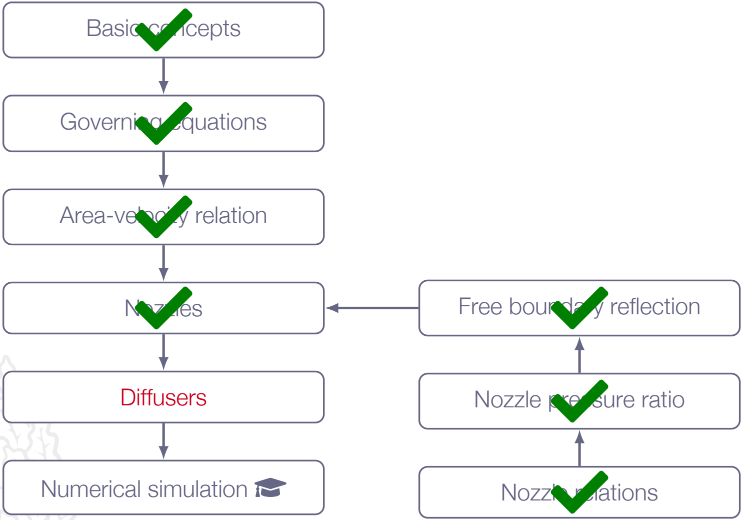
Solid-wall reflection

- Compression waves reflects as compression waves
- Expansion waves reflects as expansion waves

Free-boundary reflection

- Compression waves reflects as expansion waves
- Expansion waves reflects as compression waves

Roadmap - Quasi-One-Dimensional Flow

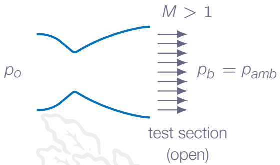


Chapter 5.5
Diffusers

Supersonic Wind Tunnel

wind tunnel with supersonic test section

open test section



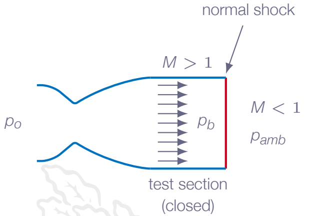
$p_o/p_b = (p_o/p_b)_{sc}$

$M = 3.0$ in test section $\Rightarrow p_o/p_b = 36.7$!!!

Supersonic Wind Tunnel

wind tunnel with supersonic test section

enclosed test section, normal shock at exit



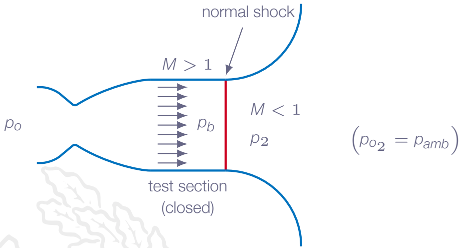
$p_o/p_{amb} = (p_o/p_b)(p_b/p_{amb}) < (p_o/p_b)_{sc}$

$M = 3.0$ in test section $\Rightarrow$
$p_o/p_{amb} = 36.7/10.33 = 3.55$

Supersonic Wind Tunnel

wind tunnel with supersonic test section

add subsonic diffuser after normal shock



$p_o/p_{amb} = (p_o/p_b)(p_b/p_2)(p_2/p_{o_2})$

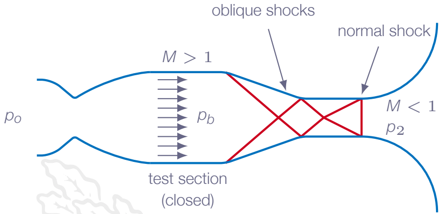
$M = 3.0$ in test section $\Rightarrow$
$p_o/p_{amb} = 36.7/10.33/1.17 = 3.04$

Note! this corresponds exactly to total pressure loss across normal shock

Supersonic Wind Tunnel

wind tunnel with supersonic test section

add supersonic diffuser before normal shock



$\left(p_{o_2} = p_{amb}\right)$

well-designed supersonic + subsonic diffuser $\Rightarrow$

1. decreased total pressure loss
2. decreased p_o and power to drive wind tunnel

Supersonic Wind Tunnel

Main problems:

1. **Complex 3D flow** in the diffuser section

   viscous effects

   complex systems of oblique shocks

   flow separation

   shock/boundary-layer interaction

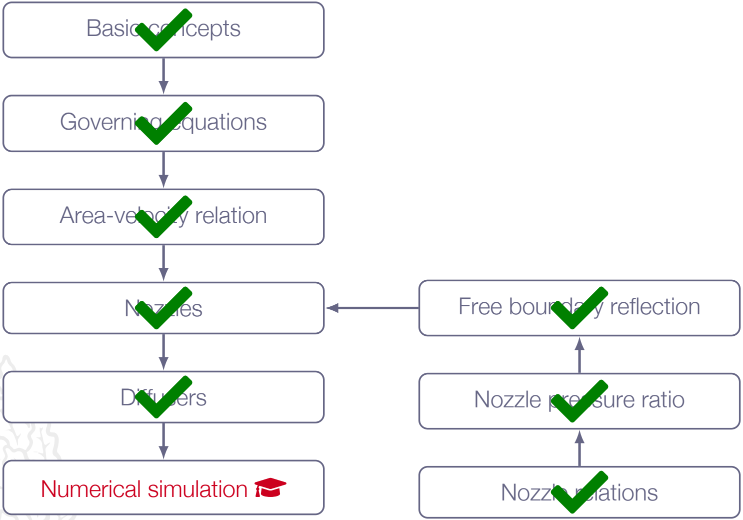
2. **Starting requirements**
   second throat must be significantly larger than first throat

   variable geometry diffuser

   second throat larger during startup procedure

   decreased second throat to optimum value after supersonic flow is established

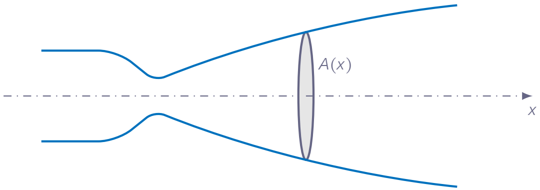
Roadmap - Quasi-One-Dimensional Flow



Quasi-One-Dimensional Euler Equations

Quasi-One-Dimensional Euler Equations

Example: choked flow through a convergent-divergent nozzle



Assumptions: inviscid, $Q = Q(x, t)$

Quasi-One-Dimensional Euler Equations

$$A(x)\frac{\partial}{\partial t}Q + \frac{\partial}{\partial x}\left[A(x)E\right] = A'(x)H$$

where $A(x)$ is the cross section area and

$$Q = \begin{bmatrix} \rho \\ \rho u \\ \rho e_o \end{bmatrix}, \; E(Q) = \begin{bmatrix} \rho u \\ \rho u^2 + p \\ \rho h_o u \end{bmatrix}, \; H(Q) = \begin{bmatrix} 0 \\ p \\ 0 \end{bmatrix}$$

Numerical Approach

Discretization:
Finite-Volume Method (FVM) - Quasi-1D formulation

Numerical scheme:
third-order characteristic upwind scheme

Time stepping technique:
three-stage second-order Runge-Kutta explicit time marching

Boundary conditions:

left-end boundary:

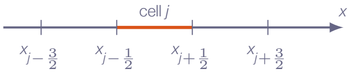
- subsonic inflow
- specify: inlet total temperature (T_o) and total pressure (p_o)

right-end boundary:

- subsonic outflow
- specify: outlet static pressure (p)

Finite-Volume Spatial Discretization

$$\left(\Delta x_j = x_{j+\frac{1}{2}} - x_{j-\frac{1}{2}}\right)$$



Integration over cell j gives:

$$\frac{1}{2}\left[A(x_{j-\frac{1}{2}}) + A(x_{j+\frac{1}{2}})\right]\Delta x_j \frac{d}{dt}\bar{Q}_j +$$
$$\left[A(x_{j+\frac{1}{2}})\hat{E}_{j+\frac{1}{2}} - A(x_{j-\frac{1}{2}})\hat{E}_{j-\frac{1}{2}}\right] =$$
$$\left[A(x_{j+\frac{1}{2}}) - A(x_{j-\frac{1}{2}})\right]\hat{H}_j$$

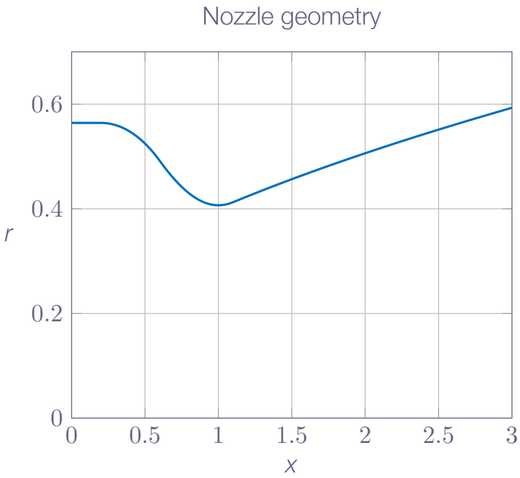
Finite-Volume Spatial Discretization

$$\bar{Q}_j = \left(\int_{x_{j-\frac{1}{2}}}^{x_{j+\frac{1}{2}}} QA(x)dx \right) \Bigg/ \left(\int_{x_{j-\frac{1}{2}}}^{x_{j+\frac{1}{2}}} A(x)dx \right)$$

$$\hat{E}_{j+\frac{1}{2}} \approx E\left(Q\left(x_{j+\frac{1}{2}}\right)\right)$$

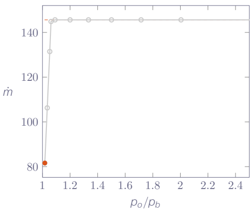
$$\hat{H}_j \approx \left(\int_{x_{j-\frac{1}{2}}}^{x_{j+\frac{1}{2}}} HA'(x)dx \right) \Bigg/ \left(\int_{x_{j-\frac{1}{2}}}^{x_{j+\frac{1}{2}}} A'(x)dx \right)$$

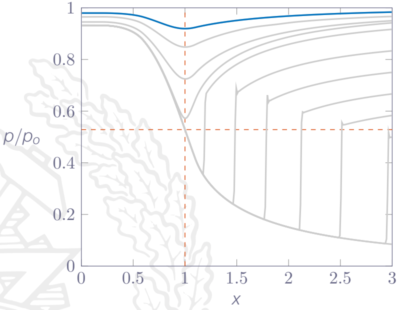
Nozzle Simulation - Back Pressure Sweep

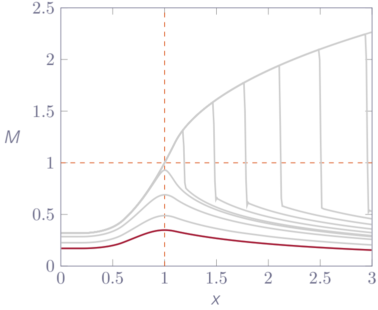


Nozzle Simulation - Back Pressure Sweep

p_o	1.20	[bar]
p_b	1.18	[bar]
p_o/p_b	1.02	
$\dot{m}$	81.61	[kg/s]
M_{max}	0.35	

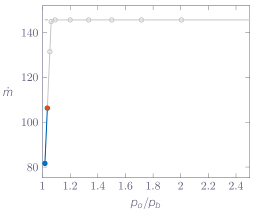


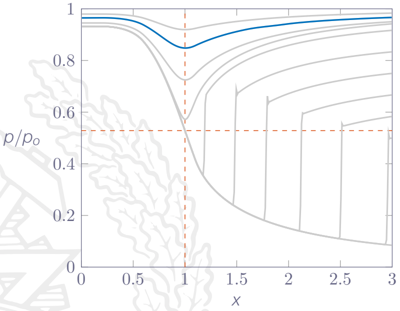


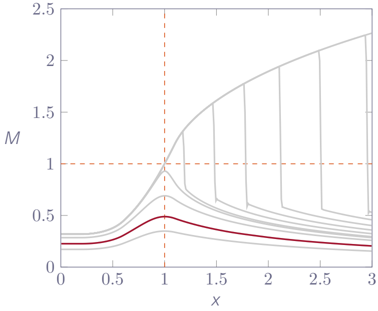


Nozzle Simulation - Back Pressure Sweep

p_o	1.20	[bar]
p_b	1.16	[bar]
p_o/p_b	1.03	
$\dot{m}$	106.27	[kg/s]
M_{max}	0.49	

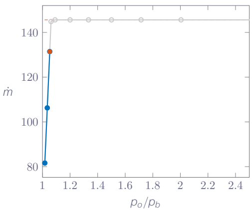


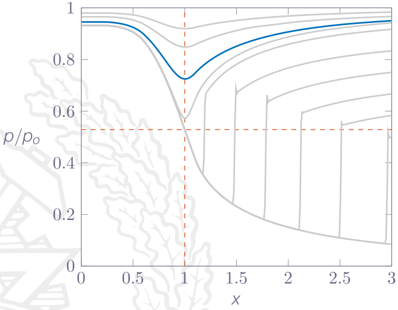


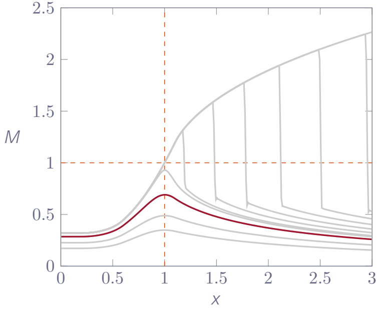


Nozzle Simulation - Back Pressure Sweep

p_o	1.20 [bar]
p_b	1.14 [bar]
p_o/p_b	1.05
$\dot{m}$	131.45 [kg/s]
M_{max}	0.69

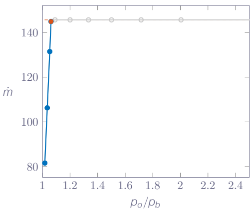


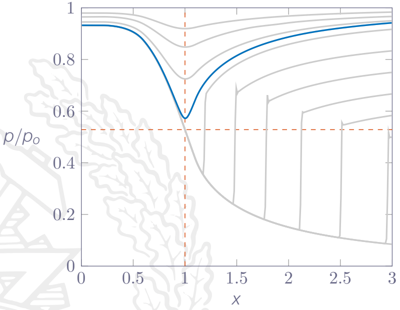


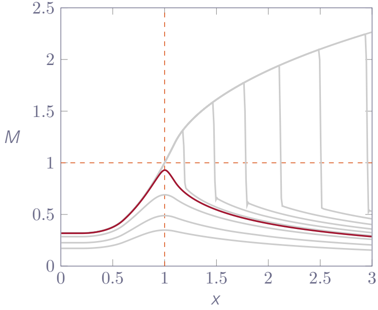


Nozzle Simulation - Back Pressure Sweep

p_o	1.20 [bar]
p_b	1.13 [bar]
p_o/p_b	1.06
$\dot{m}$	144.88 [kg/s]
M_{max}	0.93

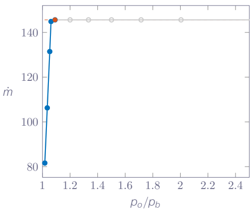


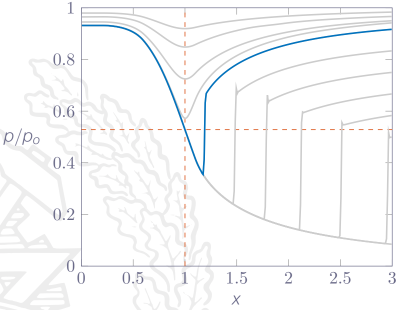


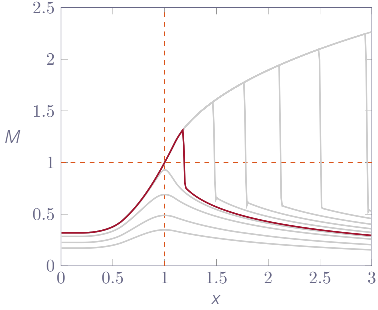


Nozzle Simulation - Back Pressure Sweep

p_o	1.20 [bar]
p_b	1.10 [bar]
p_o/p_b	1.09
$\dot{m}$	145.62 [kg/s]
M_{max}	1.31

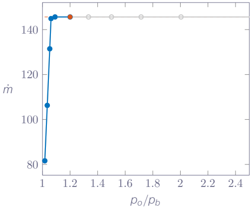


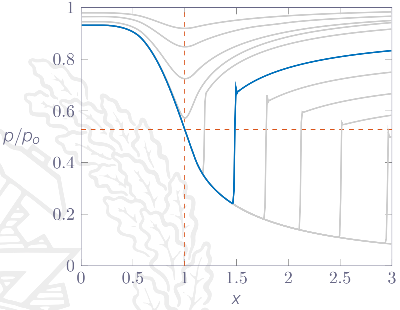


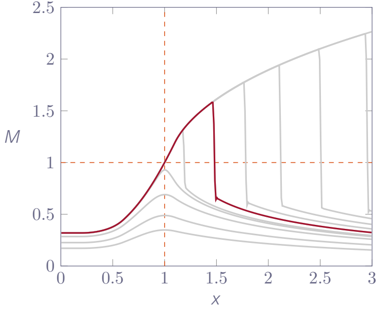


Nozzle Simulation - Back Pressure Sweep

p_o	1.20 [bar]
p_b	1.00 [bar]
p_o/p_b	1.20
$\dot{m}$	145.6 [kg/s]
M_{max}	1.58

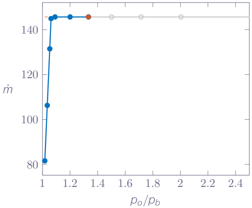


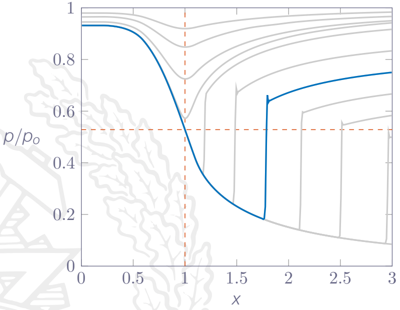


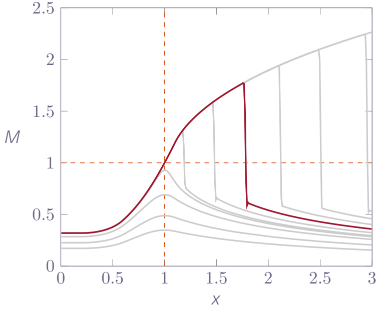


Nozzle Simulation - Back Pressure Sweep

p_o	1.20 [bar]
p_b	0.90 [bar]
p_o/p_b	1.33
$\dot{m}$	145.6 [kg/s]
M_{max}	1.77

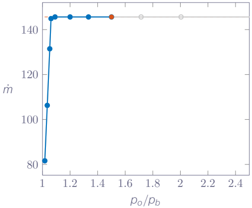


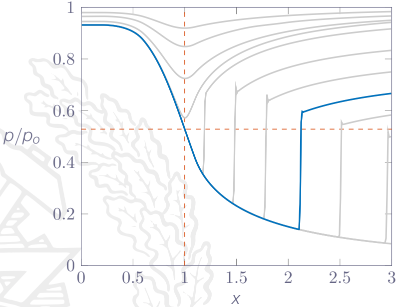


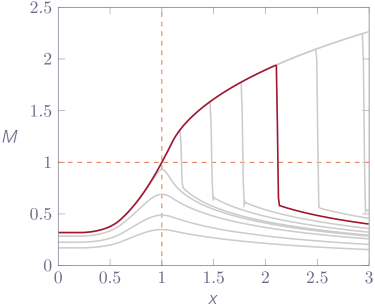


Nozzle Simulation - Back Pressure Sweep

p_o	1.20 [bar]
p_b	0.80 [bar]
p_o/p_b	1.50
$\dot{m}$	145.6 [kg/s]
M_{max}	1.94

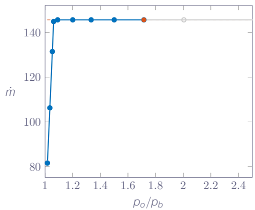


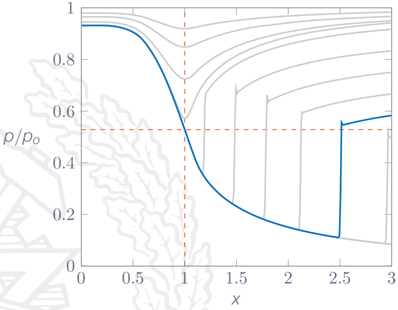


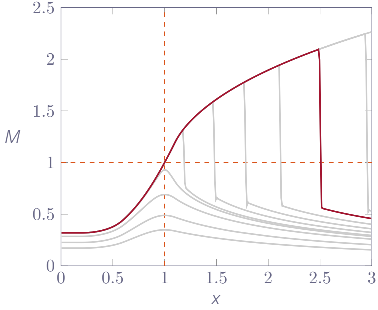


Nozzle Simulation - Back Pressure Sweep

p_o	1.20 [bar]
p_b	0.70 [bar]
p_o/p_b	1.71
$\dot{m}$	145.6 [kg/s]
M_{max}	2.10

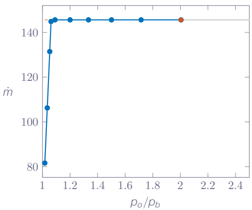


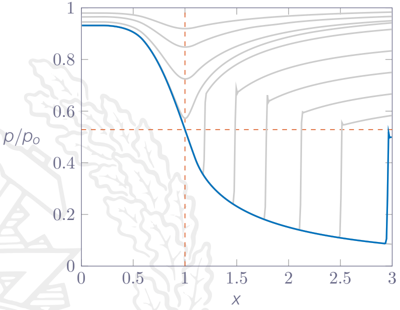


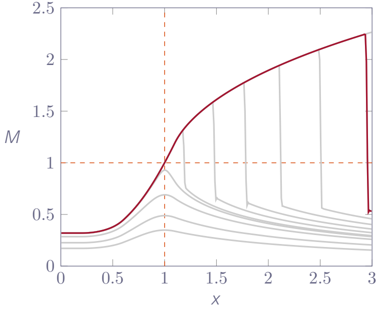


Nozzle Simulation - Back Pressure Sweep

p_o	1.20 [bar]
p_b	0.60 [bar]
p_o/p_b	2.00
$\dot{m}$	145.6 [kg/s]
M_{max}	2.24

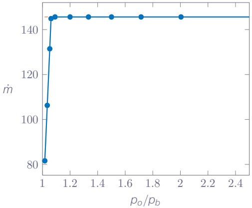


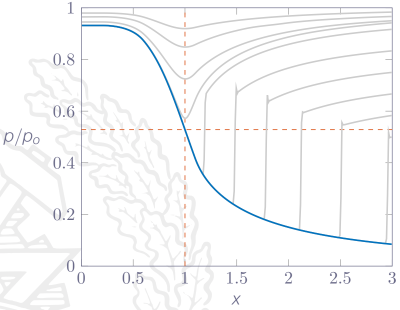


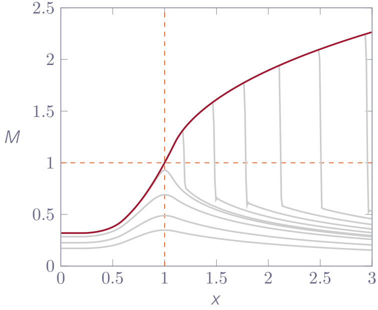


Nozzle Simulation - Back Pressure Sweep

p_o	1.20 [bar]
p_b	0.50 [bar]
p_o/p_b	11.8
$\dot{m}$	145.6 [kg/s]
M_{max}	2.26

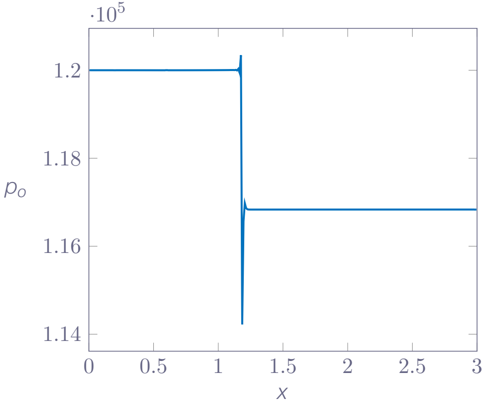






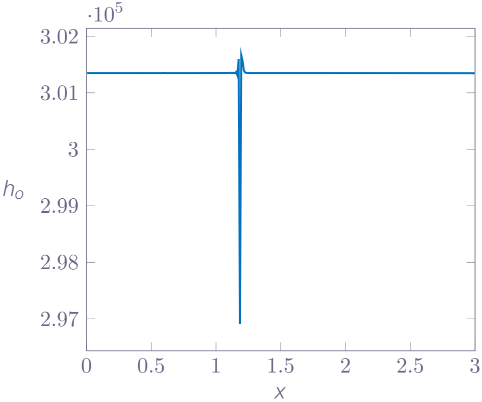
Nozzle Simulation - Back Pressure Sweep

p_o	1.20 [bar]
p_b	1.10 [bar]
p_o/p_b	1.09
$\dot{m}$	145.62 [kg/s]
M_{max}	1.31

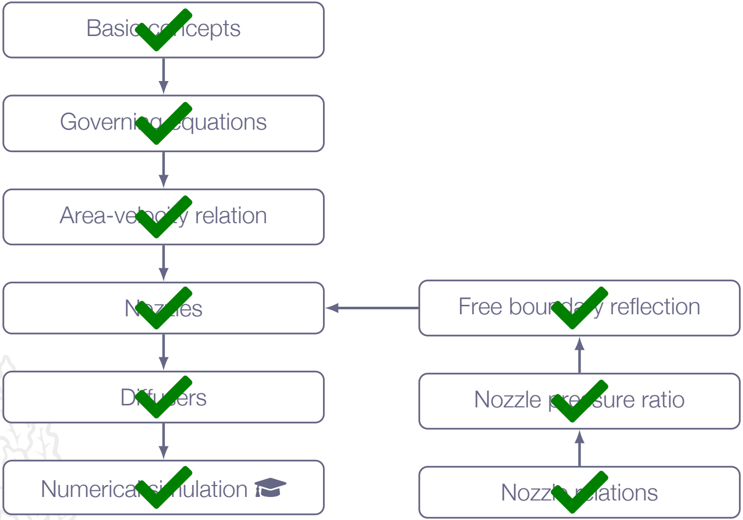


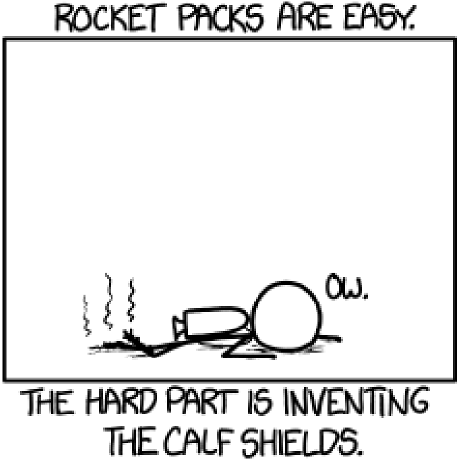
Nozzle Simulation - Back Pressure Sweep

p_o	1.20 [bar]
p_b	1.10 [bar]
p_o/p_b	1.09
$\dot{m}$	145.62 [kg/s]
M_{max}	1.31



Roadmap - Quasi-One-Dimensional Flow




ROCKET PACKS ARE EASY.
OW.
THE HARD PART IS INVENTING THE CALF SHIELDS.