



Compressible Flow TME085

One-Dimensional Steady Flow

Hugoniot Equation

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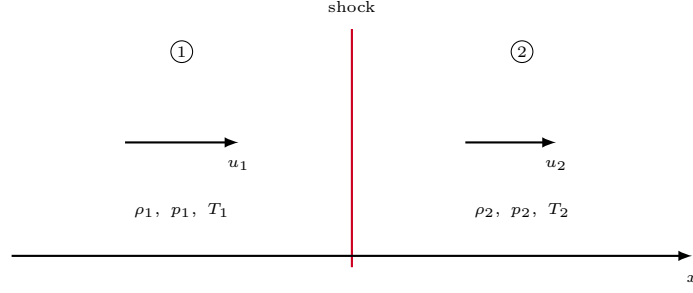


Figure 1: Stationary normal shock

Governing Equations

The starting point is to set up the governing equations for one-dimensional steady compressible flow over a control volume enclosing the normal shock (Fig. 1).

continuity:

$$\rho_1 u_1 = \rho_2 u_2 \quad (1)$$

momentum:

$$\rho_1 u_1^2 + p_1 = \rho_2 u_2^2 + p_2 \quad (2)$$

energy:

$$h_1 + \frac{1}{2}u_1^2 = h_2 + \frac{1}{2}u_2^2 \quad (3)$$

Derivation of the Hugoniot Equation

The continuity equation is rewritten and inserted into the momentum equation

$$u_1 = \left(\frac{\rho_2}{\rho_1} \right) u_2 \quad (4)$$

Replace u_1 in Eqn. 2 using Eqn. 4

$$\rho_1 \left(\frac{\rho_2}{\rho_1} \right)^2 u_2^2 + p_1 = \rho_2 u_2^2 + p_2$$

$$u_2^2 \left(\rho_1 \left(\frac{\rho_2}{\rho_1} \right)^2 - \rho_2 \right) = (p_2 - p_1)$$

$$u_2^2 \left(\left(\frac{\rho_2}{\rho_1} \right) (\rho_2 - \rho_1) \right) = (p_2 - p_1)$$

$$u_2^2 = \left(\frac{\rho_1}{\rho_2} \right) \frac{p_2 - p_1}{\rho_2 - \rho_1} \quad (5)$$

Eqn. 4 and 5 gives

$$u_1^2 = \left(\frac{\rho_2}{\rho_1} \right) \frac{p_2 - p_1}{\rho_2 - \rho_1} \quad (6)$$

Eqn. 5 and Eqn. 6 inserted in the energy equation (Eqn. 3) gives

$$h_1 + \frac{1}{2} \left(\frac{\rho_2}{\rho_1} \right) \left(\frac{p_2 - p_1}{\rho_2 - \rho_1} \right) = h_2 + \frac{1}{2} \left(\frac{\rho_1}{\rho_2} \right) \left(\frac{p_2 - p_1}{\rho_2 - \rho_1} \right) \quad (7)$$

$$h_2 - h_1 = \frac{p_2 - p_1}{2} \left[\left(\frac{\rho_2}{\rho_1} \right) \left(\frac{1}{\rho_2 - \rho_1} \right) - \left(\frac{\rho_1}{\rho_2} \right) \left(\frac{1}{\rho_2 - \rho_1} \right) \right]$$

$$h_2 - h_1 = \frac{p_2 - p_1}{2} \left[\frac{\rho_2^2 - \rho_1^2}{\rho_1 \rho_2 (\rho_2 - \rho_1)} \right] = \frac{p_2 - p_1}{2} \left[\frac{\rho_2 + \rho_1}{\rho_1 \rho_2} \right]$$

$$h_2 - h_1 = \frac{p_2 - p_1}{2} \left(\frac{1}{\rho_1} + \frac{1}{\rho_2} \right) \quad (8)$$

Now, replacing the enthalpies with internal energies using $h = e + p/\rho$ gives

$$e_2 - e_1 = \frac{p_1}{\rho_1} - \frac{p_2}{\rho_2} + \frac{p_2 - p_1}{2} \left(\frac{1}{\rho_1} + \frac{1}{\rho_2} \right)$$

which after some rewriting becomes the Hugoniot equation

$$e_2 - e_1 = \frac{p_2 + p_1}{2} \left(\frac{1}{\rho_1} - \frac{1}{\rho_2} \right) \quad (9)$$

The Hugoniot equation relates thermodynamic properties over the normal shock