



# Compressible Flow TME085

Compressible Inviscid Fluid Flow Equations

The Entropy Equation

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From the second law of thermodynamics

$$\frac{De}{Dt} = T \frac{Ds}{Dt} - p \frac{D}{Dt} \left( \frac{1}{\rho} \right) \quad (1)$$

From the energy equation on differential non-conservation form internal energy formulation

$$\frac{De}{Dt} = \dot{q} - \frac{p}{\rho} (\nabla \cdot \mathbf{v})$$

The continuity equation on differential non-conservation form

$$\frac{D\rho}{Dt} + \rho(\nabla \cdot \mathbf{v}) = 0 \Rightarrow \nabla \cdot \mathbf{v} = -\frac{1}{\rho} \frac{D\rho}{Dt}$$

and thus

$$\frac{De}{Dt} = \dot{q} + \frac{p}{\rho^2} \frac{D\rho}{Dt}$$

$$\frac{D\rho}{Dt} = -\frac{1}{\nu^2} \frac{D\nu}{Dt}$$

$$\rho \frac{De}{Dt} = \rho \dot{q} - \frac{p}{\rho \nu^2} \frac{D\nu}{Dt} = \rho \dot{q} - \rho p \frac{D\nu}{Dt}$$

$$\rho \left[ \frac{De}{Dt} + p \frac{D\nu}{Dt} - \dot{q} \right] = 0 \Rightarrow \frac{De}{Dt} = \dot{q} - p \frac{D\nu}{Dt}$$

Insert  $De/Dt$  in Eqn. 1

$$\dot{q} - p \frac{D}{Dt} \left( \frac{1}{\rho} \right) = T \frac{Ds}{Dt} - p \frac{D}{Dt} \left( \frac{1}{\rho} \right) \Rightarrow$$

$$T \frac{Ds}{Dt} = -\dot{q} \quad (2)$$

Adiabatic flow:

$$T \frac{Ds}{Dt} = 0 \tag{3}$$

In an adiabatic, steady-state, inviscid flow, entropy is constant along a streamline.