



Compressible Flow TME085

Basic Concepts

Isentropic Relations

Division of Fluid Dynamics

Department of Mechanics and Maritime Sciences

Chalmers University of Technology

Entropy

First law of thermodynamics:

$$de = \delta q - \delta w \quad (1)$$

For a reversible process: $\delta w = pd(1/\rho)$ and $\delta q = Tds$

$$de = Tds - pd\left(\frac{1}{\rho}\right) \quad (2)$$

Enthalpy is defined as: $h = e + p/\rho$ and thus

$$dh = de + pd\left(\frac{1}{\rho}\right) + \left(\frac{1}{\rho}\right) dp \quad (3)$$

Eliminate de in Eqn. 2 using Eqn. 3

$$Tds = dh - \cancel{pd\left(\frac{1}{\rho}\right)} - \left(\frac{1}{\rho}\right) dp + \cancel{pd\left(\frac{1}{\rho}\right)}$$

$$ds = \frac{dh}{T} - \frac{dp}{\rho T}$$

Using $dh = C_p T$ and the equation of state $p = \rho RT$, we get

$$ds = C_p \frac{dT}{T} - R \frac{dp}{p} \quad (4)$$

Integrating Eqn. 4 gives

$$s_2 - s_1 = \int_1^2 C_p \frac{dT}{T} - R \ln\left(\frac{p_2}{p_1}\right) \quad (5)$$

For a calorically perfect gas, C_p is constant (not a function of temperature) and can be moved out from the integral and thus

$$s_2 - s_1 = C_p \ln \left(\frac{T_2}{T_1} \right) - R \ln \left(\frac{p_2}{p_1} \right) \quad (6)$$

An alternative form of Eqn. 6 is obtained by using $de = C_v dT$ Eqn. 2, which gives

$$s_2 - s_1 = \int_1^2 C_v \frac{dT}{T} - R \ln \left(\frac{p_2}{p_1} \right) \quad (7)$$

Again, for a calorically perfect gas, we get

$$s_2 - s_1 = C_v \ln \left(\frac{T_2}{T_1} \right) - R \ln \left(\frac{p_2}{p_1} \right) \quad (8)$$

Isentropic Relations

Adiabatic and reversible processes, i.e., isentropic processes implies $ds = 0$ and thus Eqn. 6 reduces to

$$\frac{C_p}{R} \ln \left(\frac{T_2}{T_1} \right) = \ln \left(\frac{p_2}{p_1} \right)$$

$$\frac{C_p}{R} = \frac{\gamma}{\gamma - 1}$$

$$\frac{\gamma}{\gamma - 1} \ln \left(\frac{T_2}{T_1} \right) = \ln \left(\frac{p_2}{p_1} \right) \Rightarrow$$

$$\frac{p_2}{p_1} = \left(\frac{T_2}{T_1} \right)^{\gamma/(\gamma-1)} \quad (9)$$

In the same way, Eqn. 8 gives

$$\frac{\rho_2}{\rho_1} = \left(\frac{T_2}{T_1} \right)^{1/(\gamma-1)} \quad (10)$$

Eqn. 11 and Eqn. 10 constitutes the isentropic relations

$$\frac{p_2}{p_1} = \left(\frac{\rho_2}{\rho_1} \right)^\gamma = \left(\frac{T_2}{T_1} \right)^{\gamma/(\gamma-1)} \quad (11)$$